

# Joker Games for checking History-Determinism

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## 1 History-deterministic parity automata

**History-determinism:** Automata where non-determinism can be resolved 'on-the-fly'.

- More succinct than deterministic parity automata [5]
- Algorithmically efficient for verification [4]
- Admit compositionality with games (Also known as good-for-games automata [4])

## 2 What are history-deterministic automata?

History-deterministic automata are those on which Eve wins the corresponding history-determinism game:

- Adam selects letter  $a_i$
- Eve selects transition  $q_i \xrightarrow{a_i} q_{i+1}$

**Eve's winning condition:** Eve's run is accepting if Adam's word is accepting.

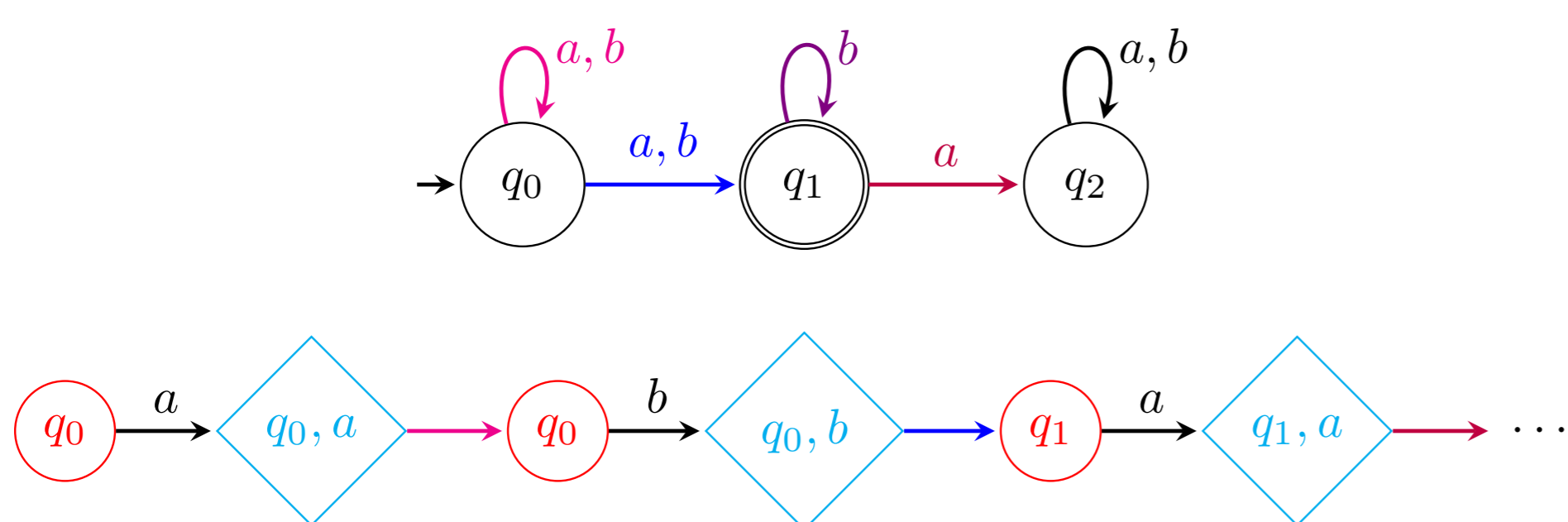


Figure 1: A non-deterministic Büchi automaton and a play of history-determinism game on it.

## 3 Algorithms for checking history-determinism

Solving the HD game directly takes exponential time as it requires determinisation of the automaton. A potential solution is the two-token game, where Adam constructs two runs 'on-the-fly' as well, along with Eve.

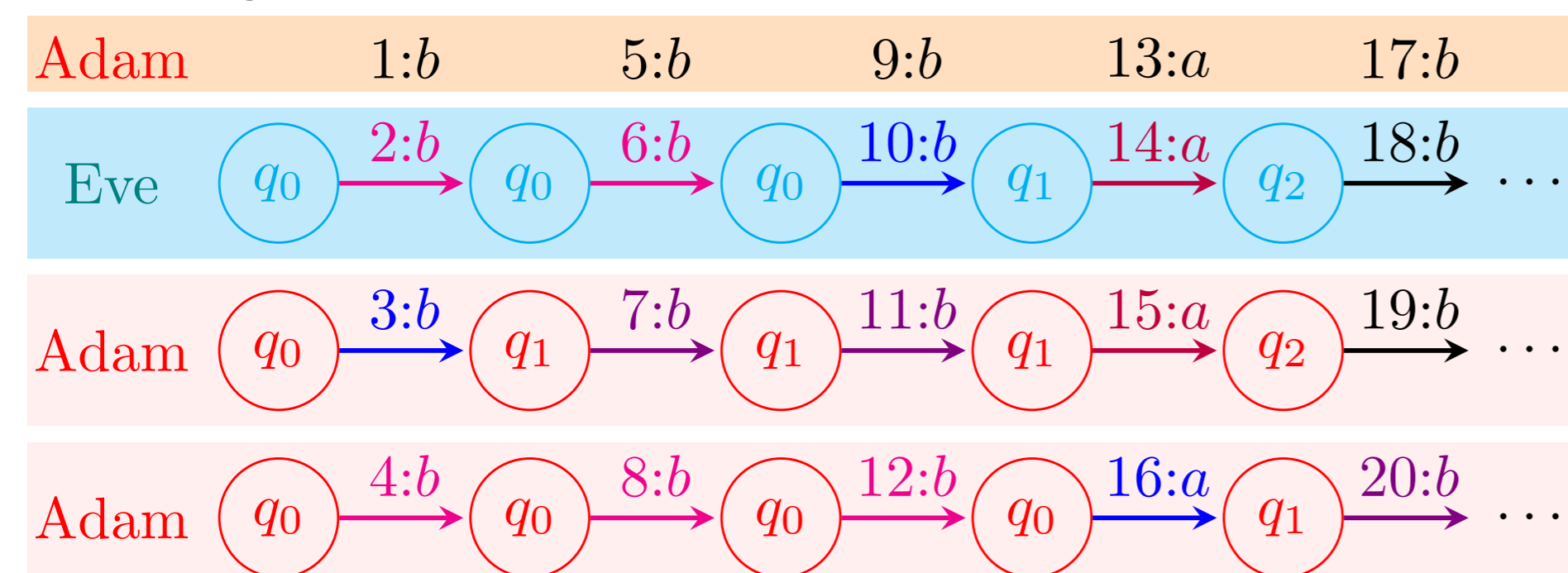


Figure 2: The figure shows a play of the two-token game on the automaton depicted in Figure 1. The numbers indicate the order of moves in the game.

**Eve's winning condition:** Eve's run is accepting if one of Adam's run is accepting.

Eve wins the two-token game on a Büchi or a co-Büchi automaton if and only if it is history-deterministic [2, 3].

**2-token conjecture:** Eve wins the two-token game on a parity automaton if and only if it is history-deterministic.

## 4 When one-token game is enough

**Theorem 1 ([1]).** One-token games characterise history-determinism on semantically-deterministic Büchi automata.

Semantically deterministic automata: all non-deterministic choices lead to language equivalent states.

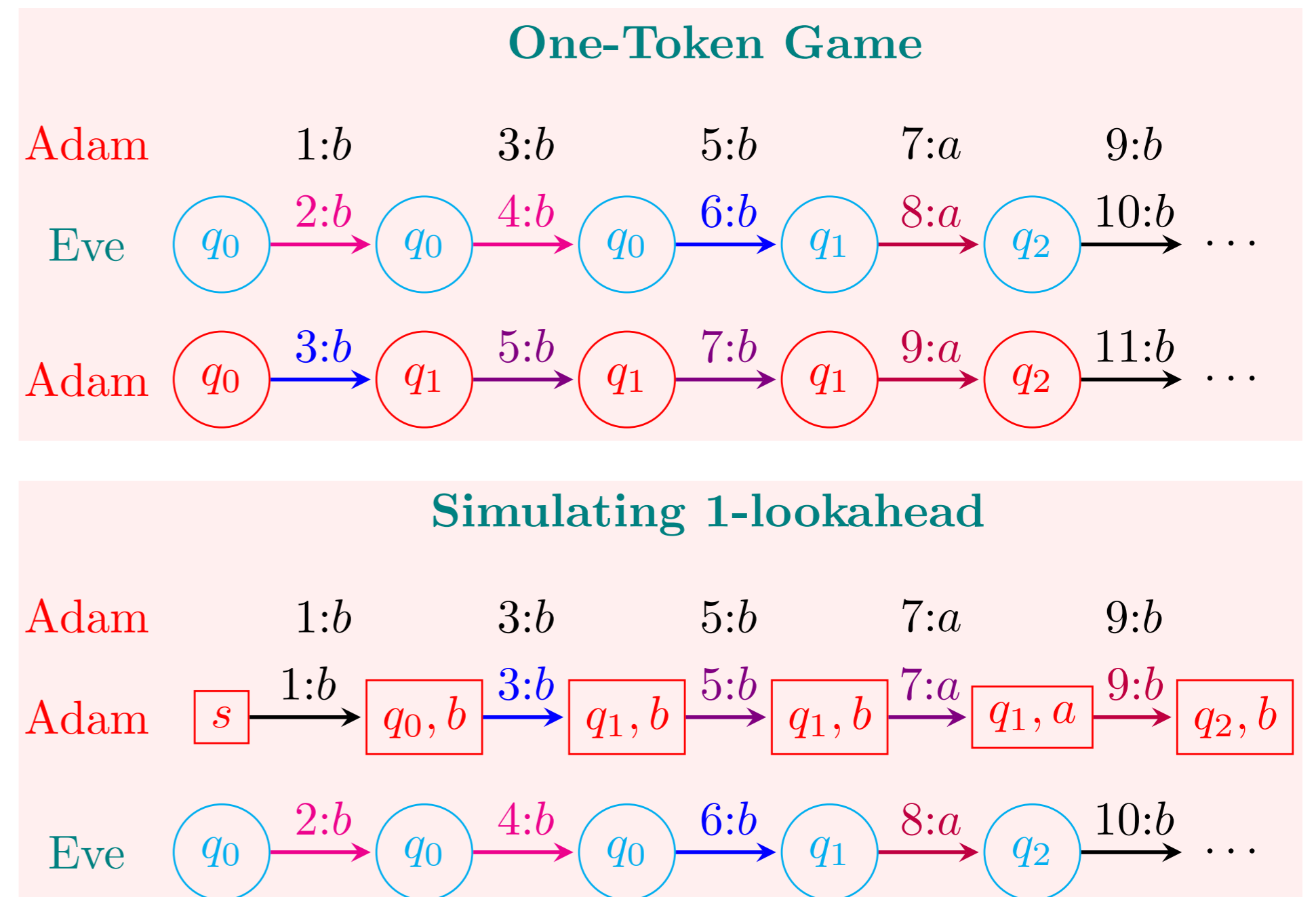


Figure 3: The power of one-token game:  
 Eve wins one-token game on an automaton  $A$   
 $\Leftrightarrow A$  simulates ' $A$  with 1-lookahead'  
 $\Leftrightarrow A$  simulates ' $A$  with  $k$ -lookahead' for any  $k \geq 0$

**Corollary 1** (of Theorem 1, also Theorem 20 in [2]). Two-token games characterise history-determinism on Büchi automaton.

**1-token conjecture:** One-token game characterises history-determinism on semantically deterministic parity automata.

- 1-token conjecture is open for parity and co-Büchi automata
- 1-token conjecture implies the 2-token conjecture

## 5 Determinisation of HD Büchi automata

**Theorem 2 ([1]).** History-deterministic Büchi automata can be determinised in polynomial time with a quadratic state-space blowup.

This solves an open problem from 2015 of Kuperberg and Skrzypczak, where they gave a non-deterministic polynomial time determinisation procedure [5].

**Open:** Is the quadratic state-space blowup necessary? Are HD-Büchi automata more succinct than deterministic Büchi automata?

## References

- [1] Joker games for checking history-determinism. *to appear*, 2023.
- [2] Marc Bagnol and Denis Kuperberg. Büchi Good-for-Games Automata Are Efficiently Recognizable. In *FSTTCS*, 2018.
- [3] Udi Boker, Denis Kuperberg, Karoliina Lehtinen, and Michał Skrzypczak. On the Succinctness of Alternating Parity Good-For-Games Automata. In *FSTTCS*, 2020.
- [4] Thomas A. Henzinger and Nir Piterman. Solving games without determinization. In *CSL*, 2006.
- [5] Denis Kuperberg and Michał Skrzypczak. On determinisation of good-for-games automata. In *ICALP*, 2015.