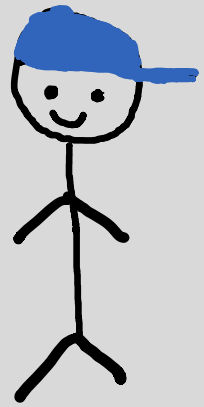
History-Deterministic One-Counter Nets

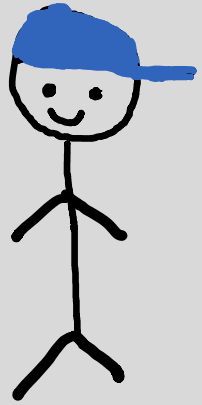
Aditya Prakash, K. S. Thejaswini

Environment



System

Environment

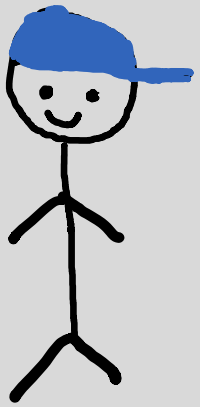


System

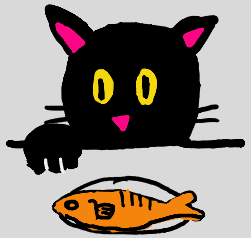
Instruction

Action

Environment



1. x n

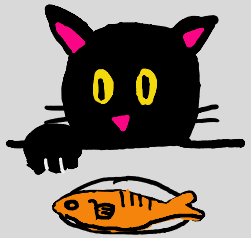


x n

2. Play x m

x m

1. x n

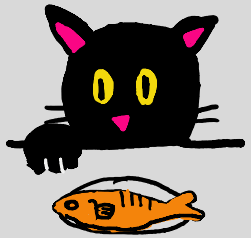
 , +1 x n

2. Play x m

, -1 x m

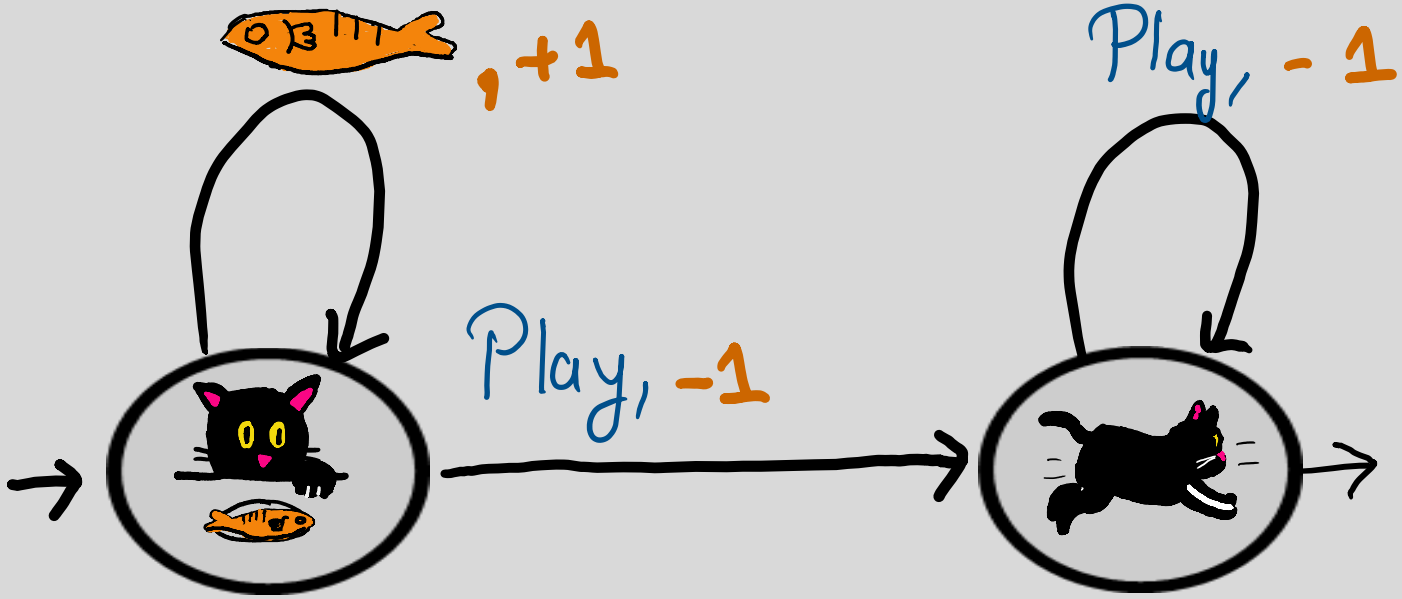
$n \geq m$

1. x n

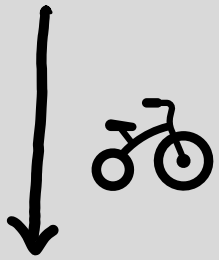
 , +1 x n

2. Play x m

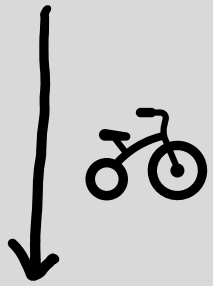
, -1 x m



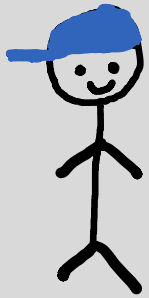
1.



2. Park 1



3. Park 2

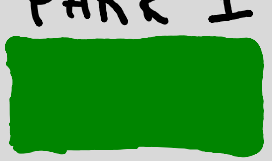


1.

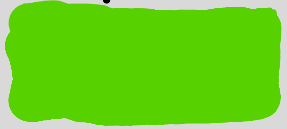
2. Park 1

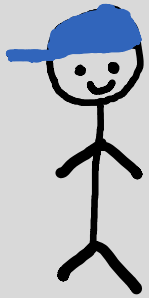
3. Park 2

PARK 1



PARK 2





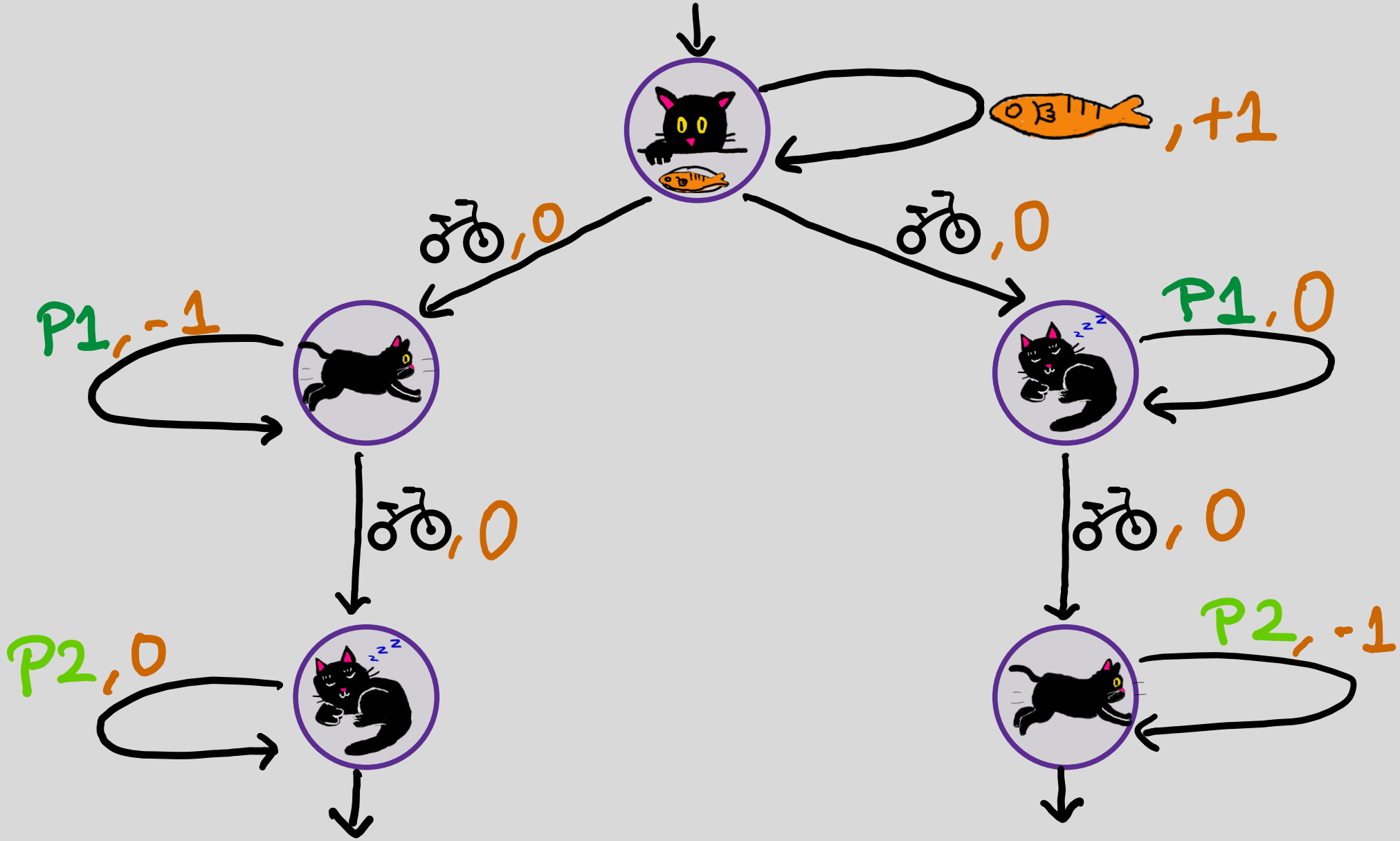
1.

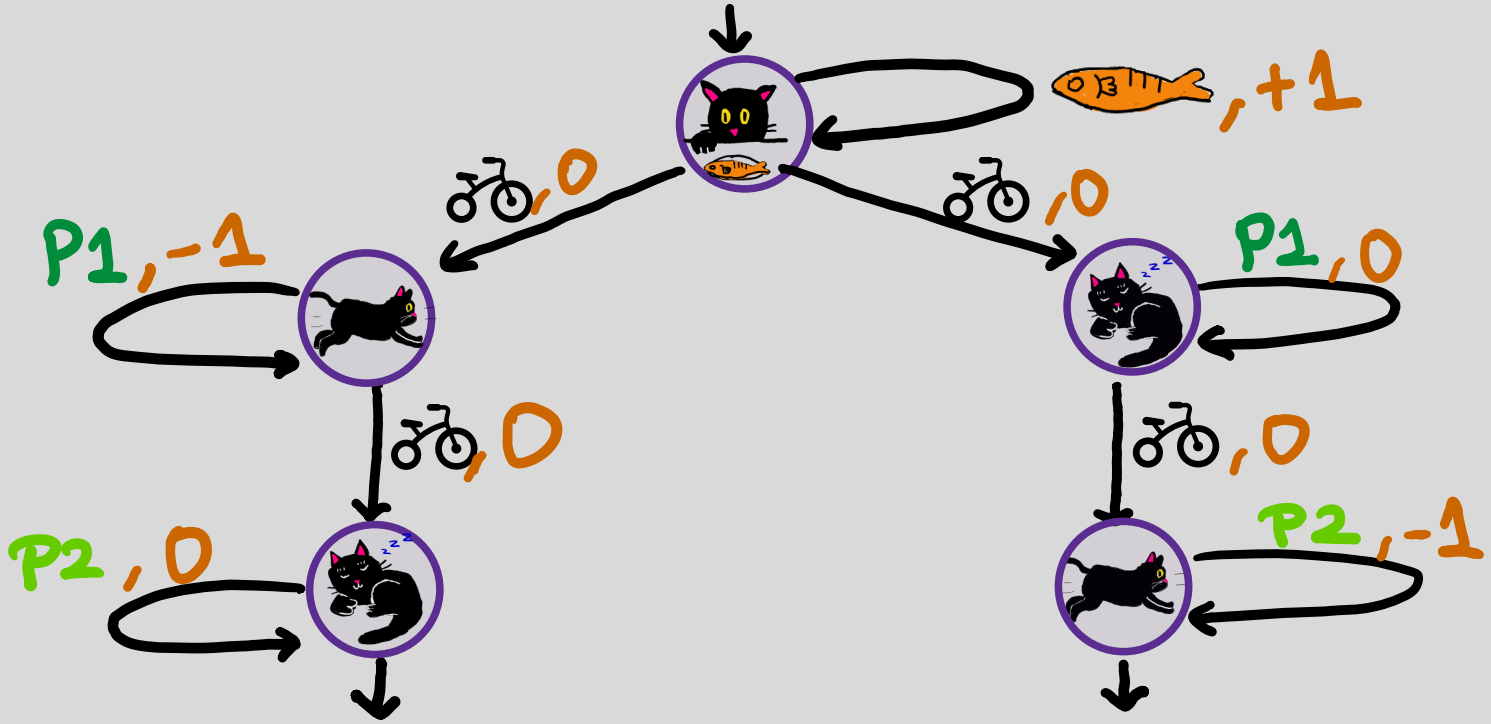
2. Park 1

3. Park 2

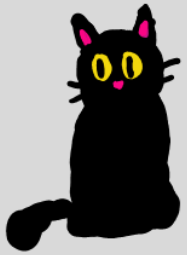
PARK 1

PARK 2


,+1
,0
,0
P1,-1
P1,0
,0
,0
P2,0
P2,-1

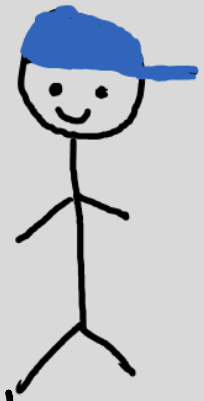


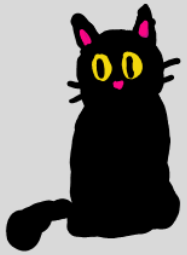
$$L = \{ \text{🐟}^{i} \text{🚲} P1^{j} \text{🚲} P2^{k} \mid i \leq j \text{ or } i \leq k \}$$



(with 5)

I will spend
4 hours in
Park 1,
7 hours in
Park 2.

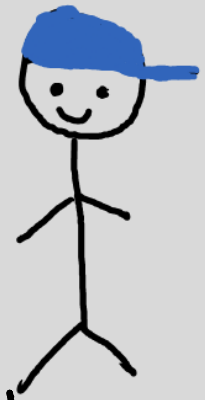


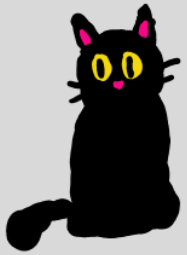


(with 5)

I will spend
4 hours in
Park 1,
7 hours in
Park 2.

I will play in
Park 1.





(with 5)

I will spend
4 hours in
Park 1,
7 hours in
Park 2.

I will play in
Park 1.

(with 5)

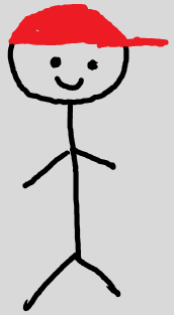
I will spend
4 hours in
Park 1,
3 hours in
Park 2.

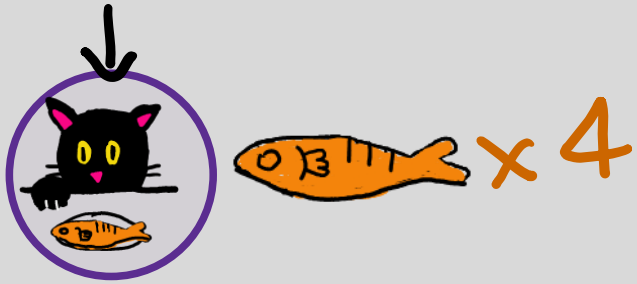
(with 5
I will spend
4 hours in
Park 1,
7 hours in
Park 2.
I will play in
Park 1.

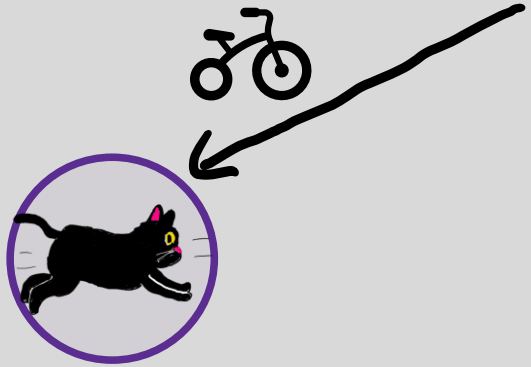
(with 5
I will spend
4 hours in
Park 1,
3 hours in
Park 2.
I can play in
either parks!!

Tec

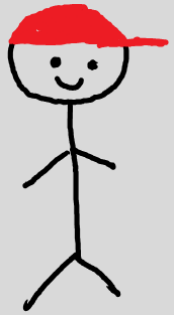
Tec







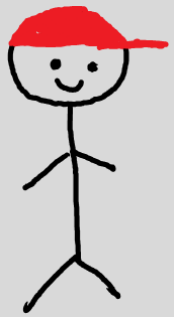
Tec



x 4

P1 x 5

Tec

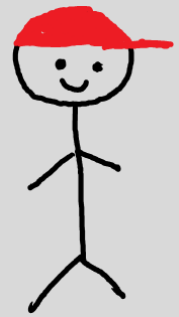


x 4

P1 x 3

P1 x 5

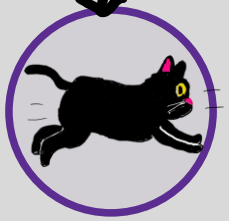
Tec



x 4

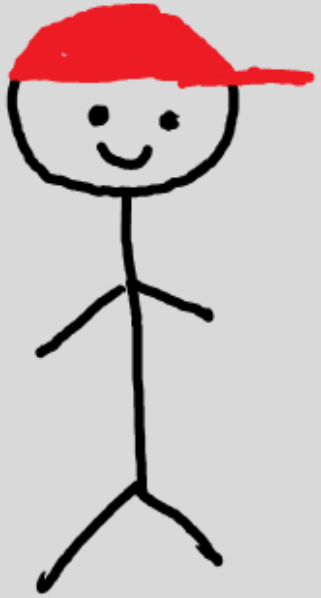
P1 x 5

P1 x 3

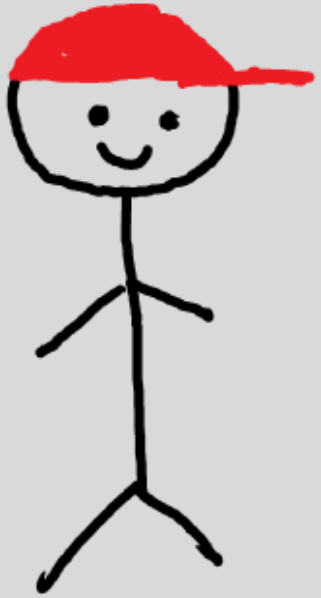


P2 x 7

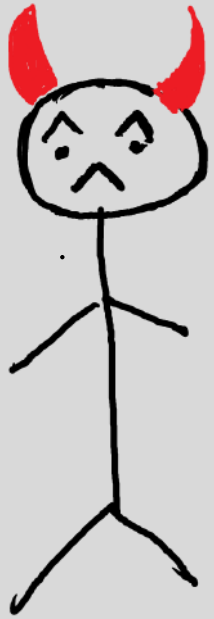
Tec



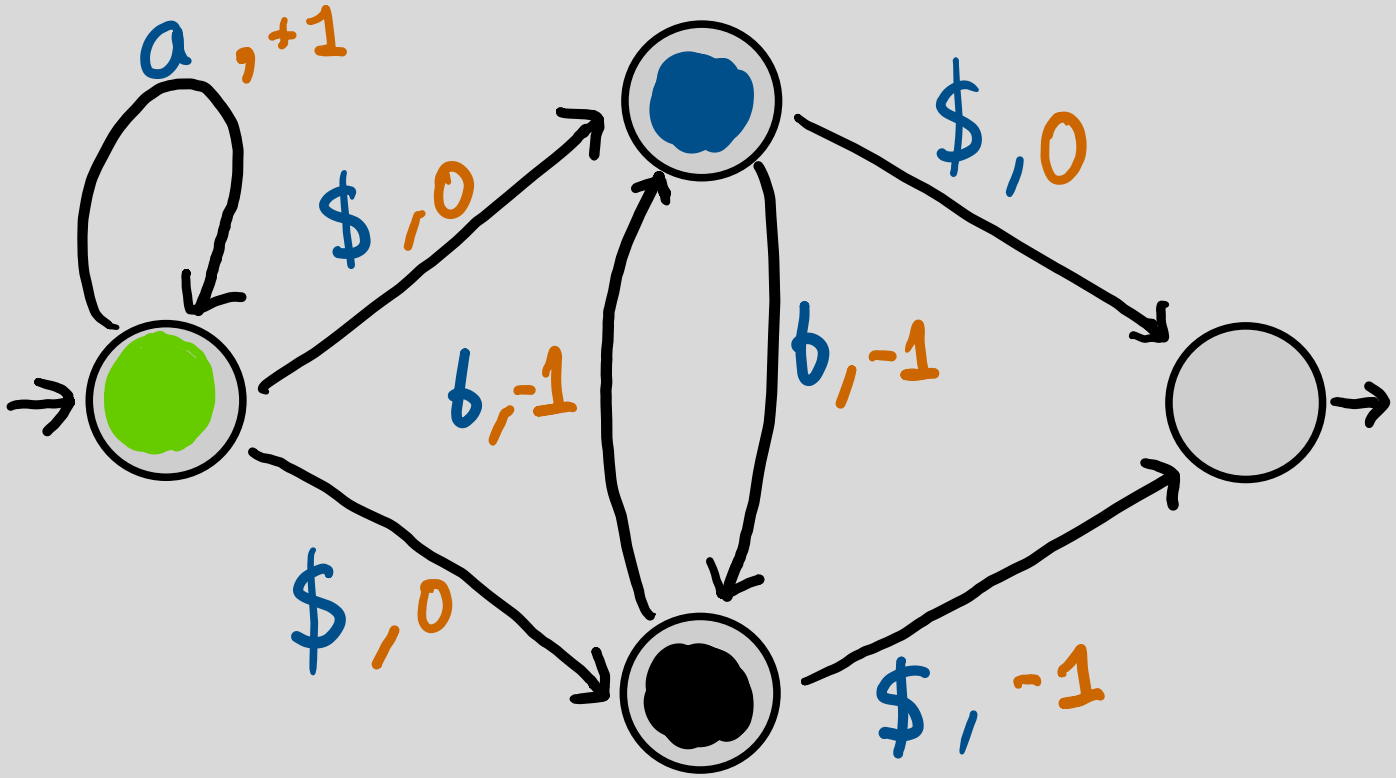
Tec

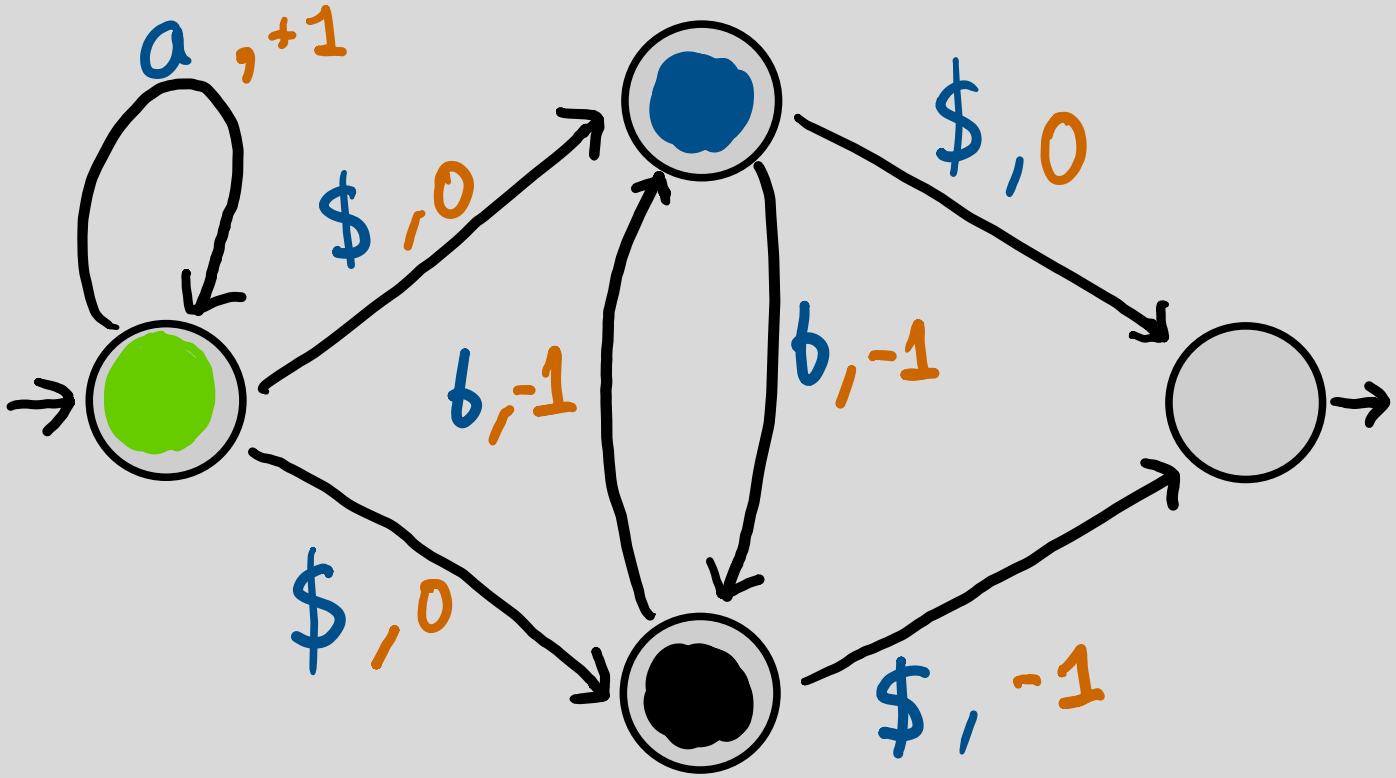


The Evil Cat sitter

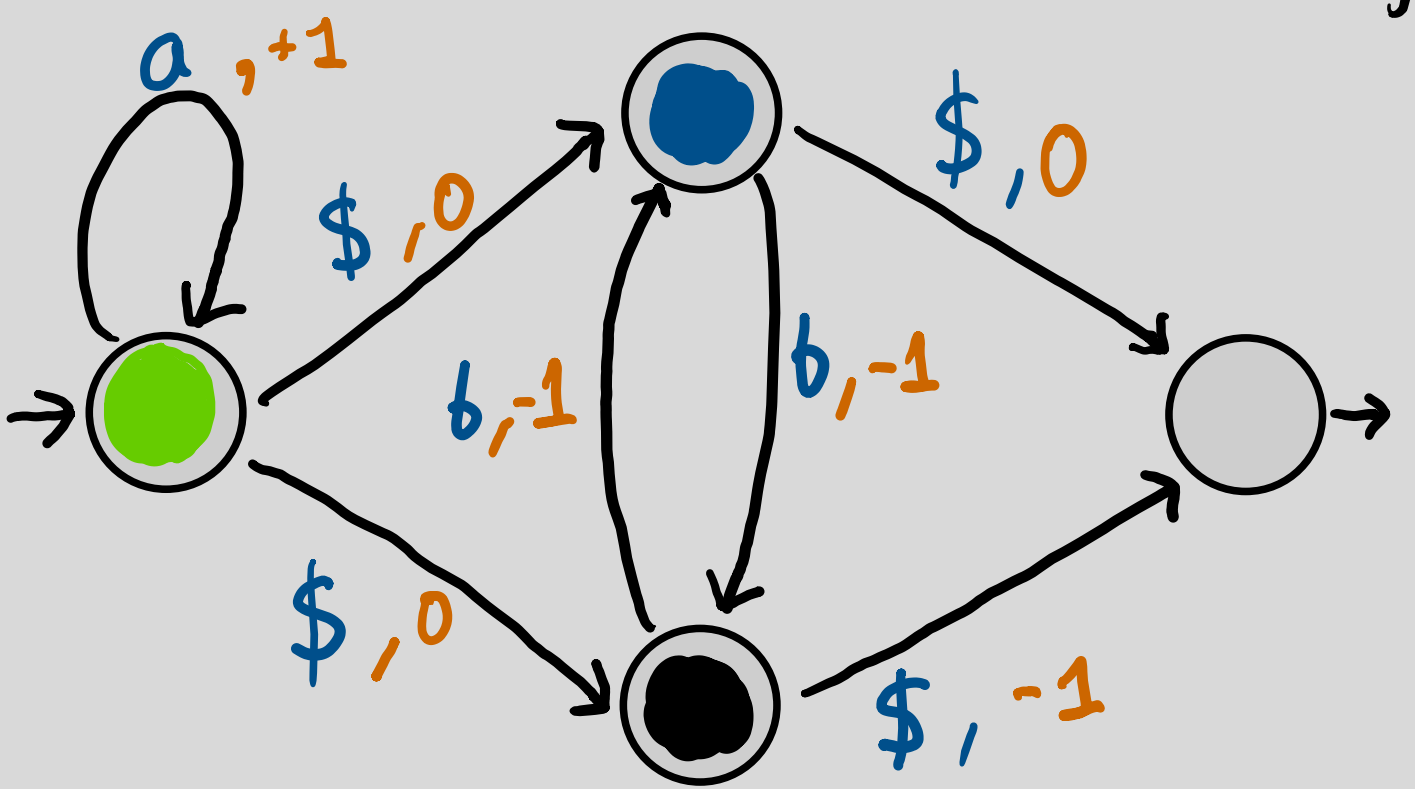


Definition: We say an one-counter net is history-deterministic if has a strategy that produces an accepting run whenever gives an accepting word.


a ,+1
$,0
b,-1
b,-1
$,0
$,0
$,-1

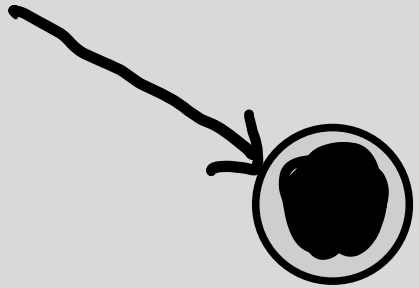


$$L = \{ a^n \$ b^m \$ \mid n \geq m \}$$

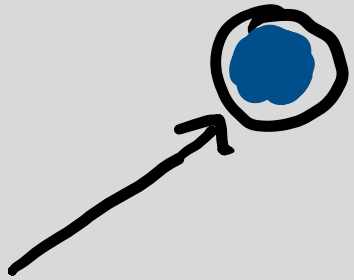


At ● , on $:

Counter = 2n+1



Counter = 2n



$$L = \{ a^n \$ b^m \$ \mid n \geq m \}$$

Theorem: Given an history-deterministic OCN,

1. There is an eventually periodic strategy for that is effectively computable.

Theorem: Given an history-deterministic OCN,

1. There is an eventually periodic strategy for that is effectively computable.

Corollary: Every history-deterministic OCN can be converted to a deterministic OCA.

Theorem: Checking whether a given OCN is history-deterministic is PSPACE-complete.

Open Problems

We showed HD-OCN $\subset$ DOCA.

a. Are HD-OCA as expressive as DOCA.

Open Problems

We showed HD-OCN $\subset$ DOCA.

a. Are HD-OCA as expressive as DOCA.

b. Is HD-OCN = OCN $\cap$ DOCA?

Open Problems

We showed HD-OCN $\subset$ DOCA.

a. Are HD-OCA as expressive as DOCA.

b. Is HD-OCN $=$ OCN $\cap$ DOCA?

Do all HD-pushdown automata have Turing-computable resolvers?

[Guha, Jecker, Lehtinen, Zimmermann '21]