

History - Deterministic One-Counter Nets

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Deterministic Models

Algorithmically efficient,
Better closure properties

Non-deterministic Models

Succinct, Expressive

Deterministic
Models

Algorithmically efficient,
Better closure properties

History-determinism

Non-deterministic
Models

Succinct, Expressive

History-Deterministic One-Counter Nets

- ▶ What is history-determinism?
- ▶ The resolvers in an HD-OCN are semilinear
Corollary: Every HD-OCN can be converted to a deterministic one-counter automaton.
- ▶ Complexity of checking history-determinism.

1. History - Determinism

System



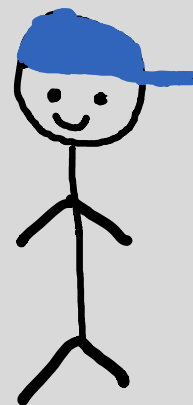
Instruction



Action



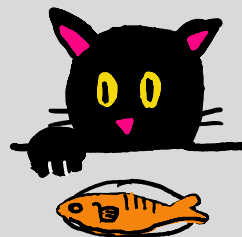
Environment



1.

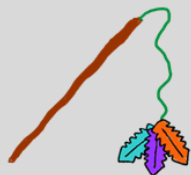


$\times n$



$\times n$

2.

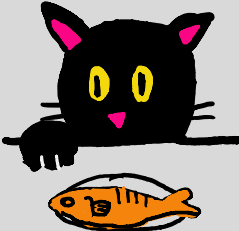


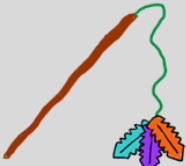
$\times m$



$\times m$

1.  $\times n$

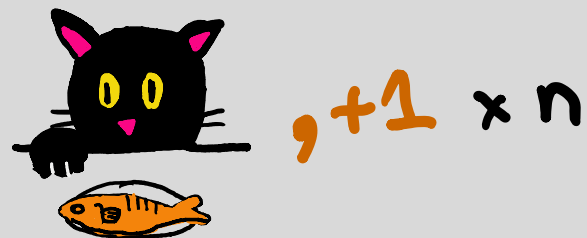
 , $+1 \times n$

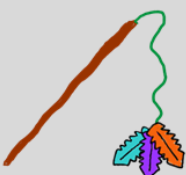
2.  $\times m$

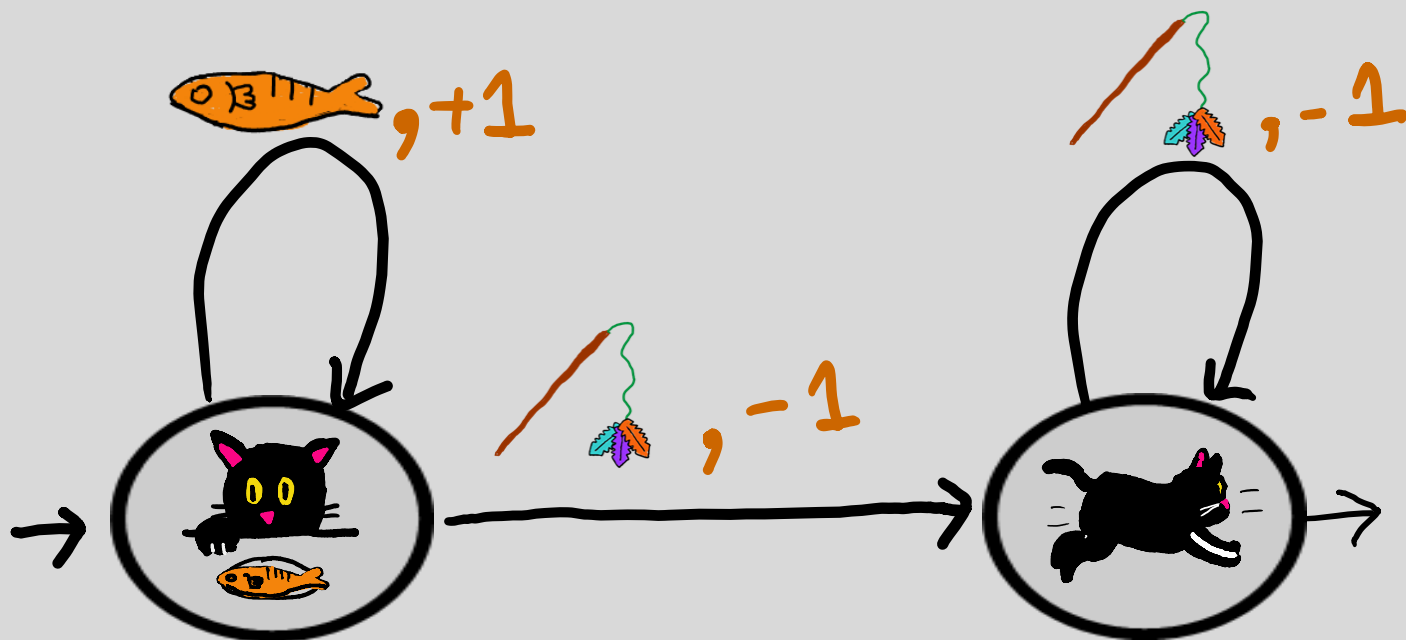
 , $-1 \times m$

$$n \geq m \geq 1$$

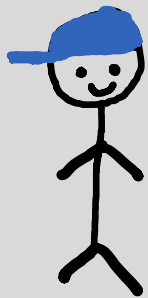
1.  $\times n$



2.  $\times m$



One-Counter
Net



1.



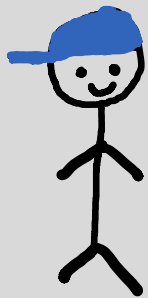
2.

Park 1



3.

Park 2



1.



2.

Park 1



3.

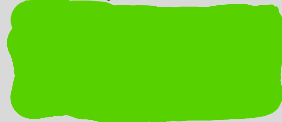
Park 2

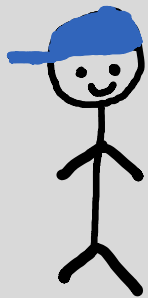


PARK 1



PARK 2





1.



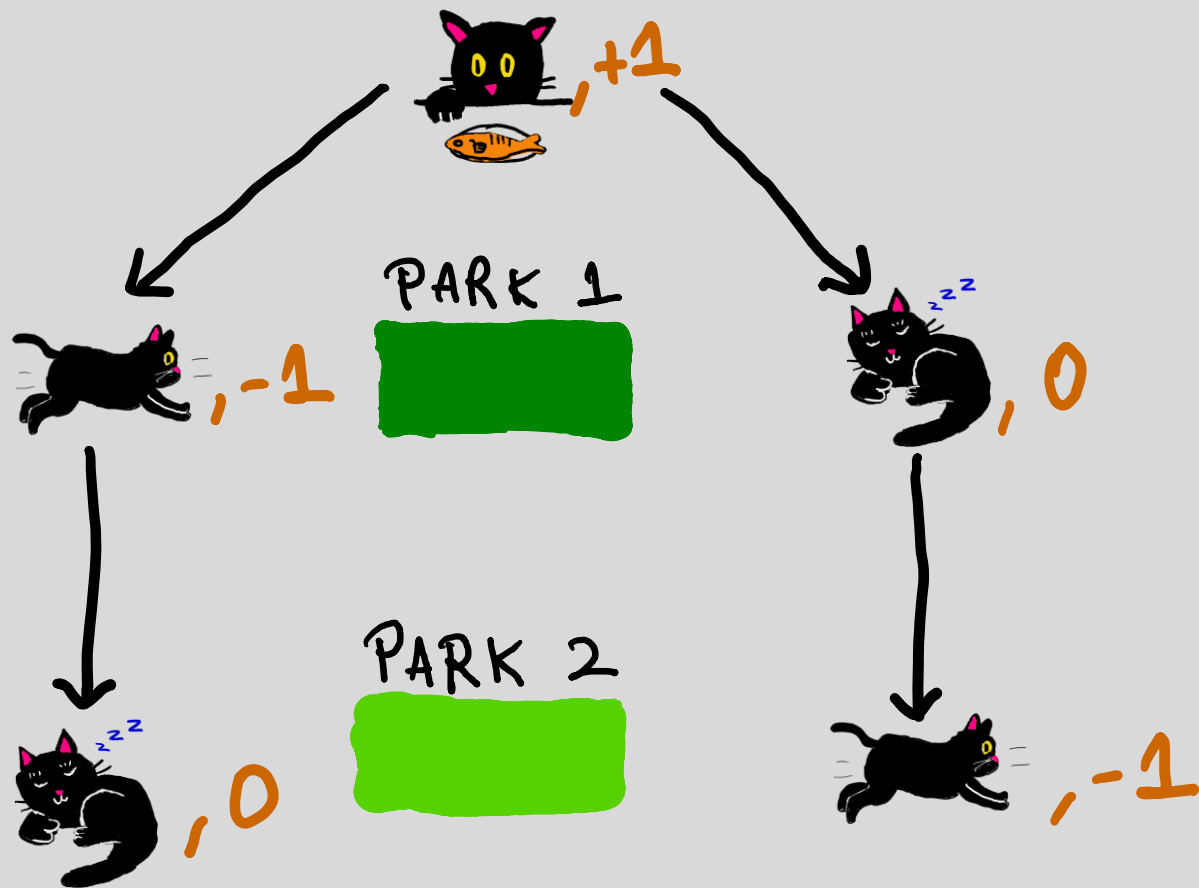
2.

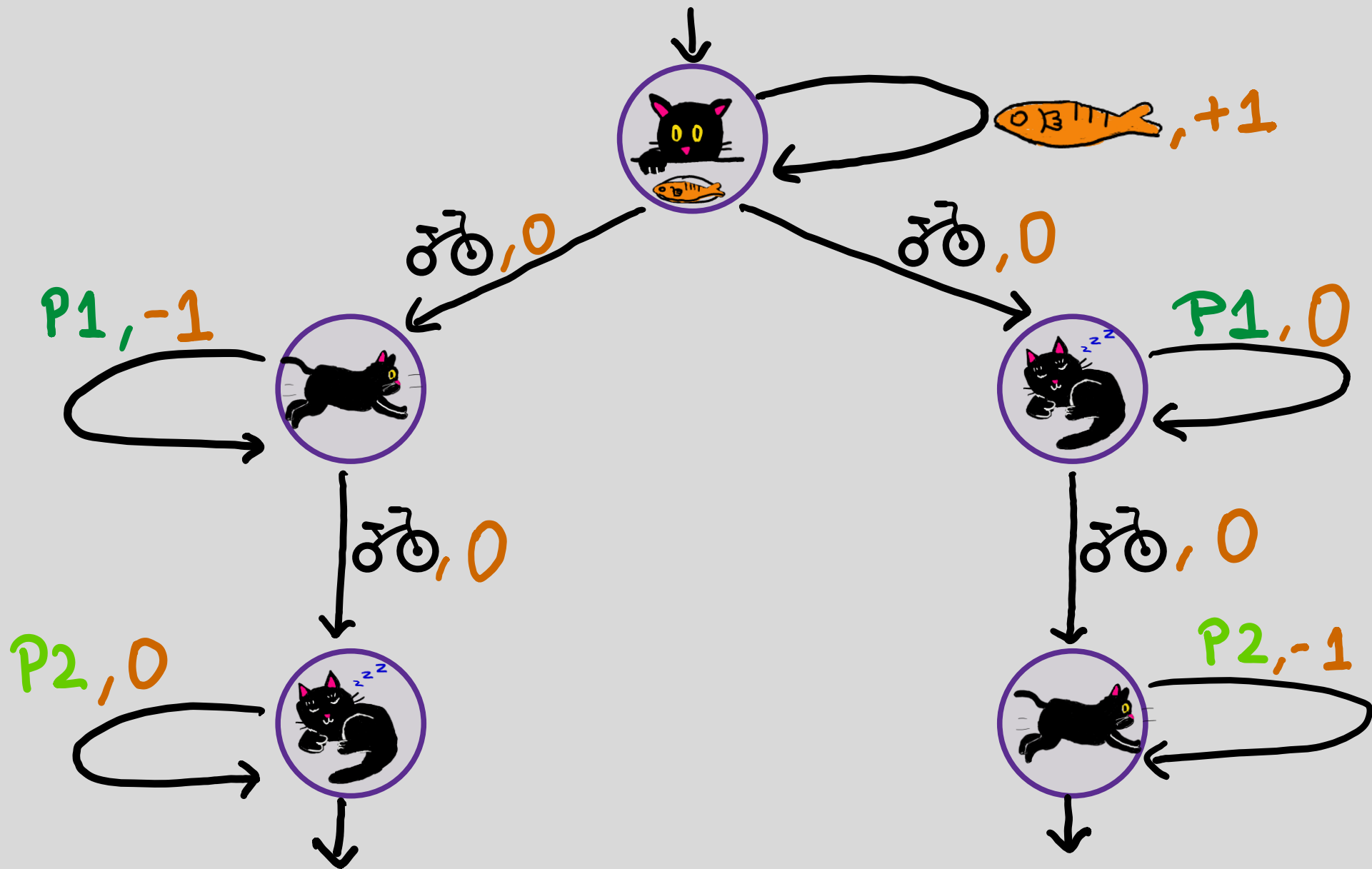
Park 1

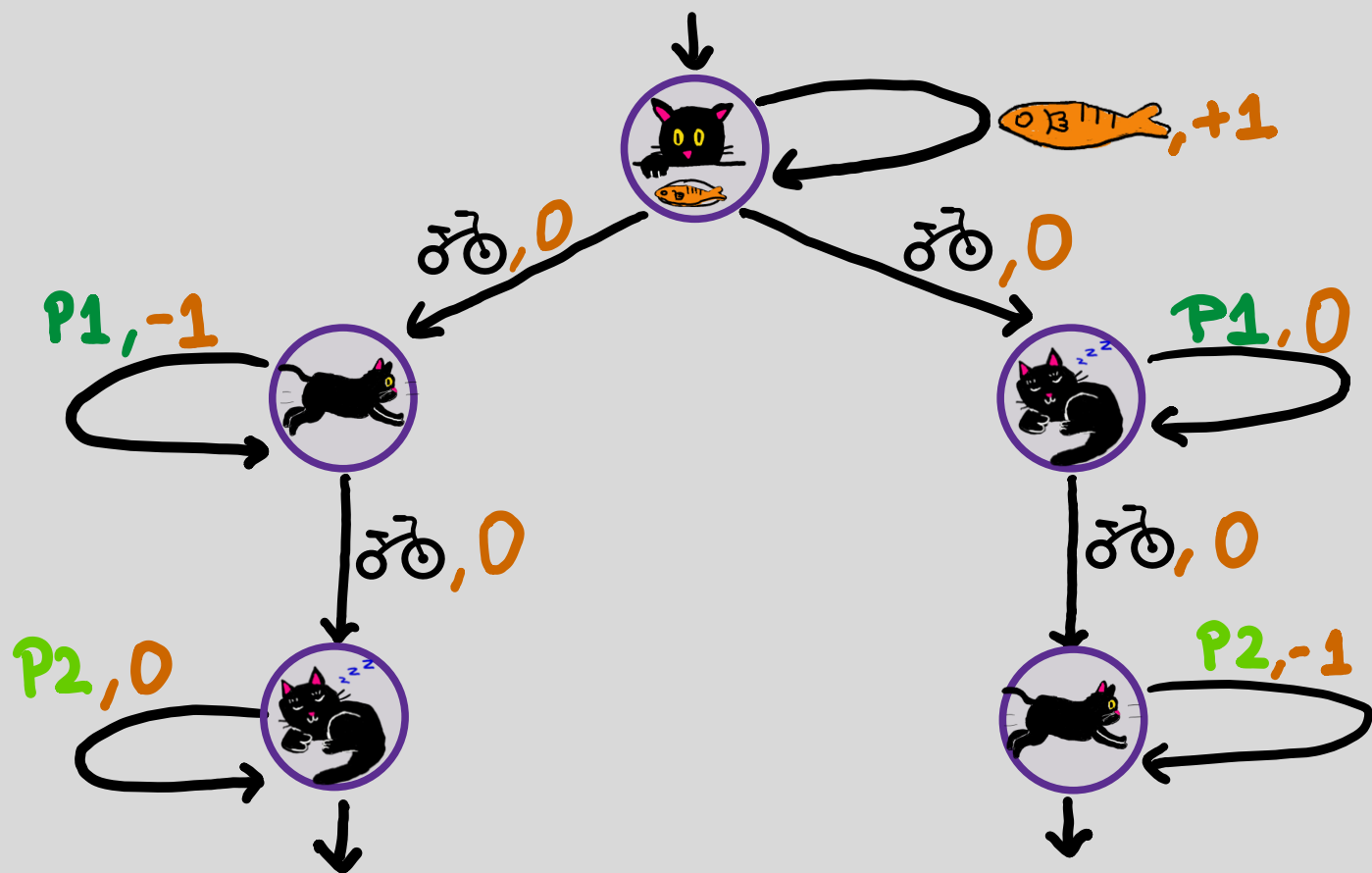


3.

Park 2







$$L = \left\{ \text{fish}^i \text{ bicycle } P1^j \text{ bicycle } P2^k \mid i \geq j \text{ or } i \geq k \right\}$$



(with 5 )

I will spend 4 hours
in Park 1, 7 hours
in Park 2.



(with 5 )



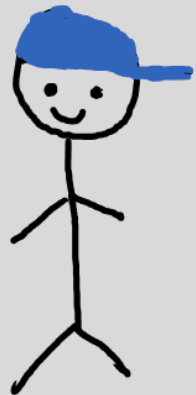
I will spend 4 hours
in Park 1, 7 hours
in Park 2.



OK, I will
play in Park 1.

(with 5 )

I will spend 4 hours
in Park 1, 7 hours
in Park 2.

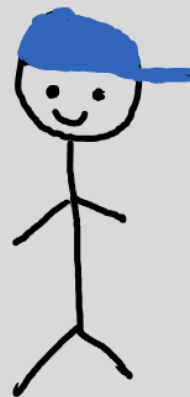


OK, I will
play in Park 1.



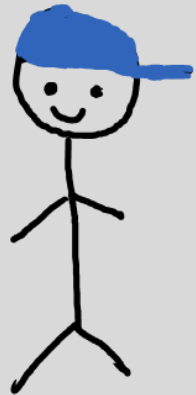
(with 5 )

I will spend 4 hours
in Park 1, 3 hours
in Park 2.



(with 5 )

I will spend 4 hours
in Park 1, 7 hours
in Park 2.



OK, I will
play in Park 1.



(with 5 )

I will spend 4 hours
in Park 1, 3 hours
in Park 2.



I can play
in either parks!!

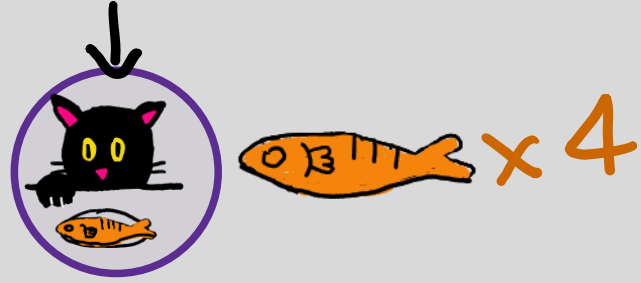


(with 5 )

Tec



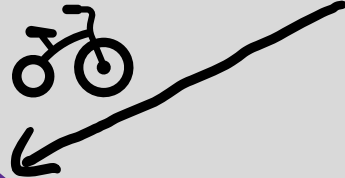
Tec



Tec



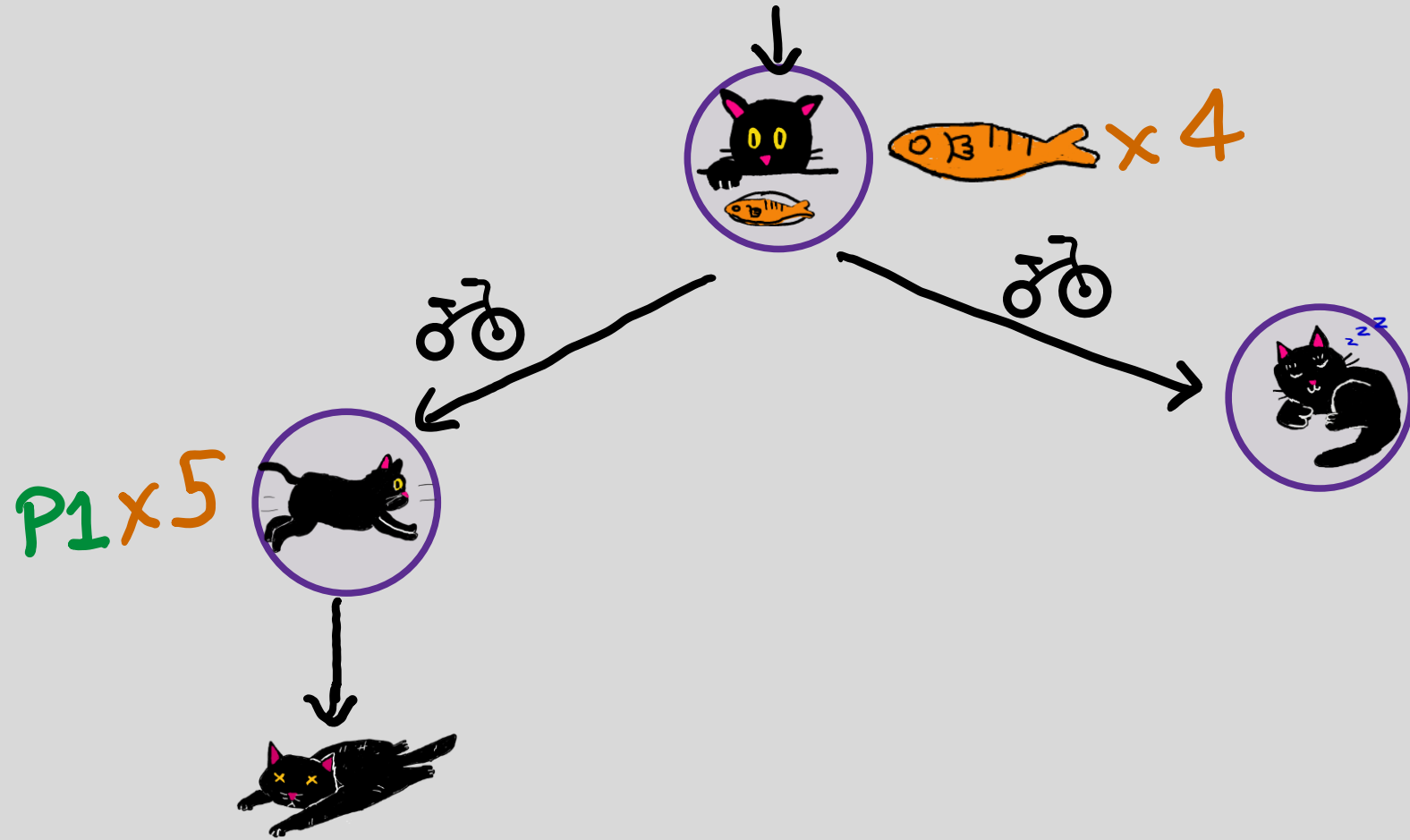
Tec



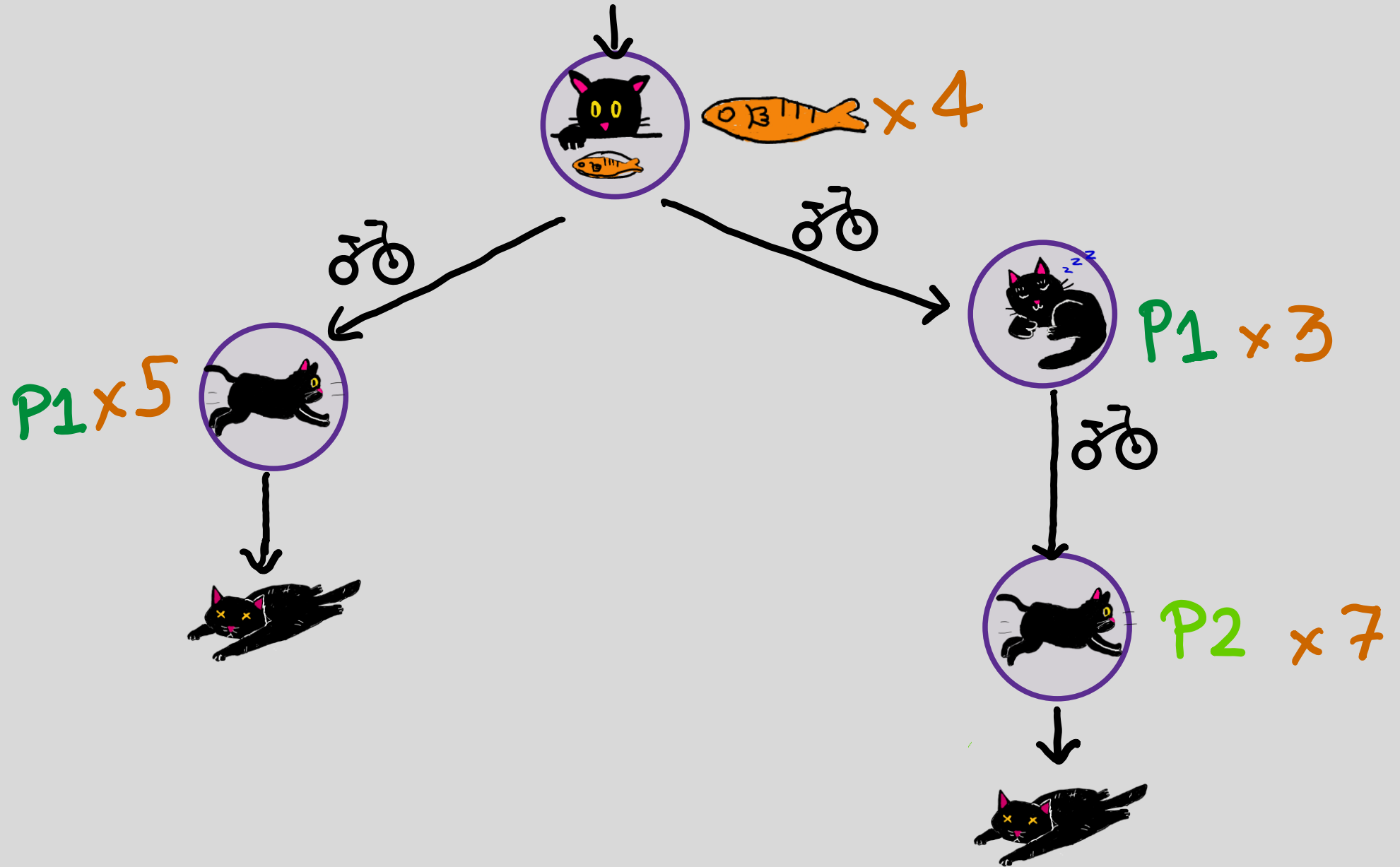
$P1 \times 5$



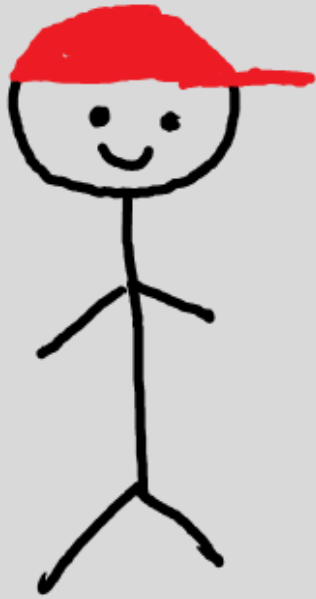
Tec



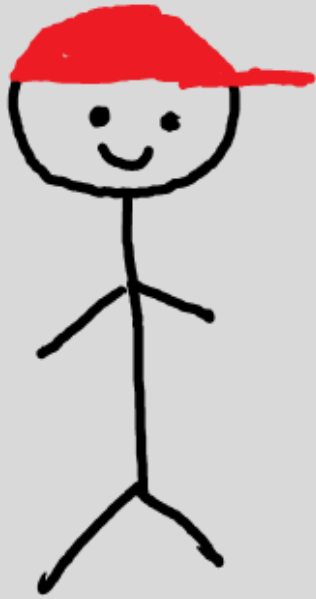
Tec



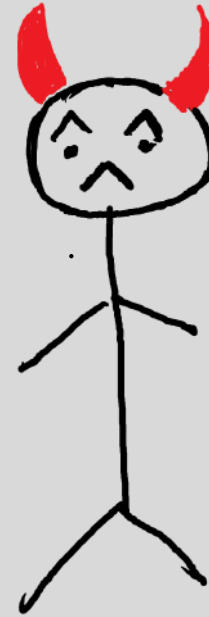
Tec



Tec



The Evil
Cat sitter



Definition: We say an one-counter net is

history-deterministic if  has a strategy
that produces an accepting run whenever

 gives an accepting word.

History - Determinism Game:

History - Determinism Game:

Starts at ( , 0):

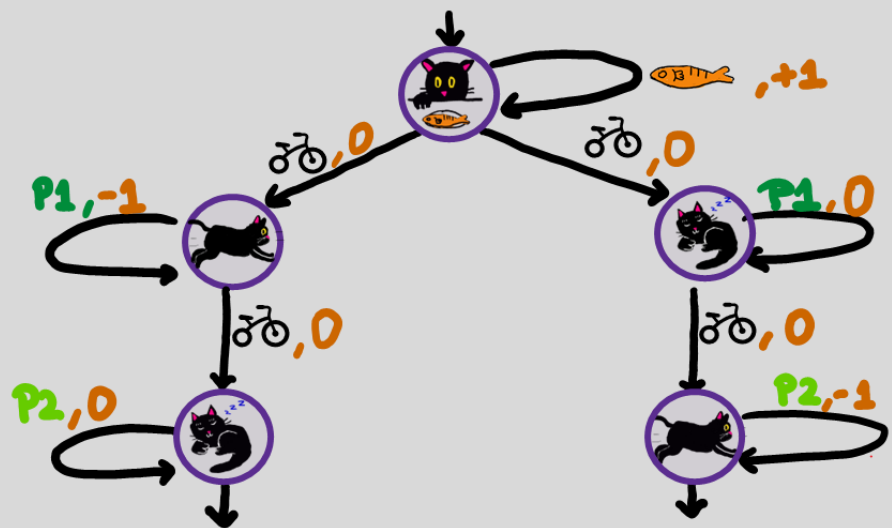
1.  selects letter
2.  selects transition

History - Determinism Game:

Starts at (, 0):

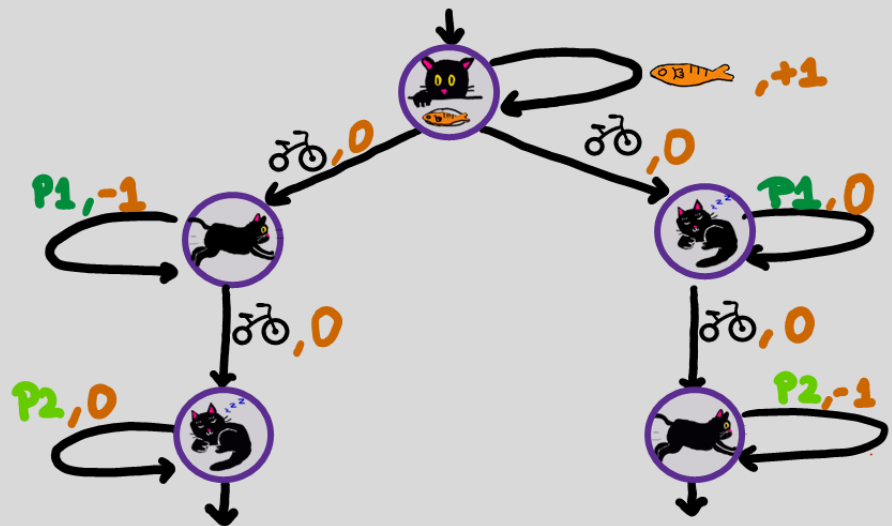
1.  selects letter
2.  selects transition

Winning Condition for : If 's word is accepting,
and 's run is rejecting.

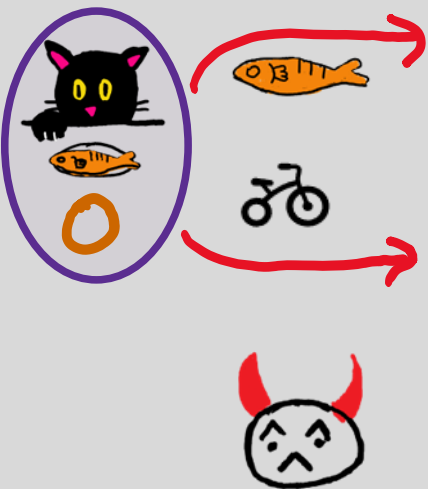


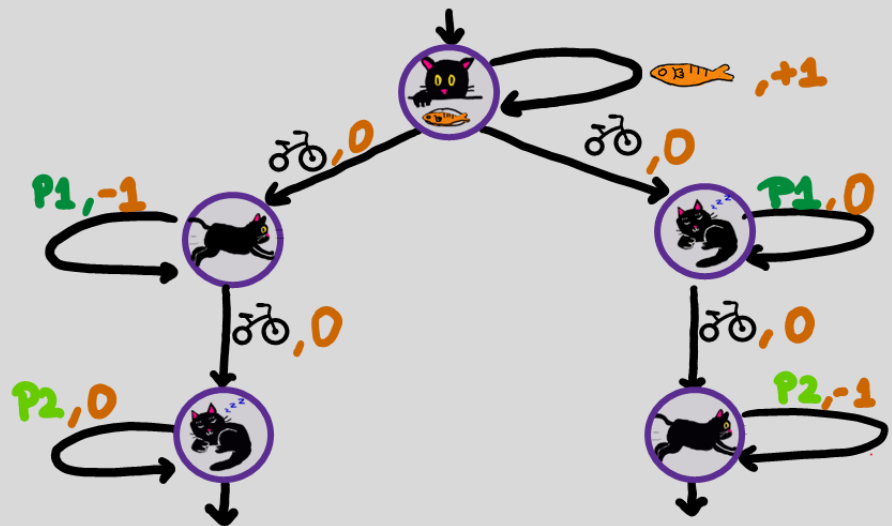
$$L = \{ \text{fish}^i \text{ bicycle}^{P1^j} \text{ bicycle}^{P2^k} \mid i \geq j \text{ or } i \geq k \}$$



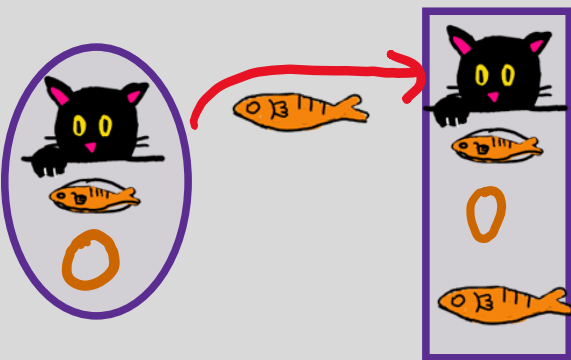


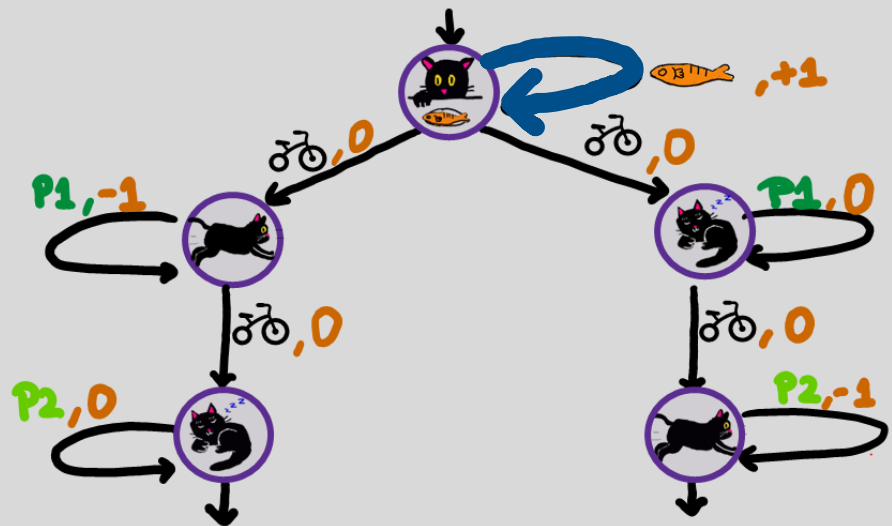
$$L = \{ \text{fish}^i \text{ } \text{P1}^j \text{ } \text{P2}^k \mid i \geq j \text{ or } i \geq k \}$$



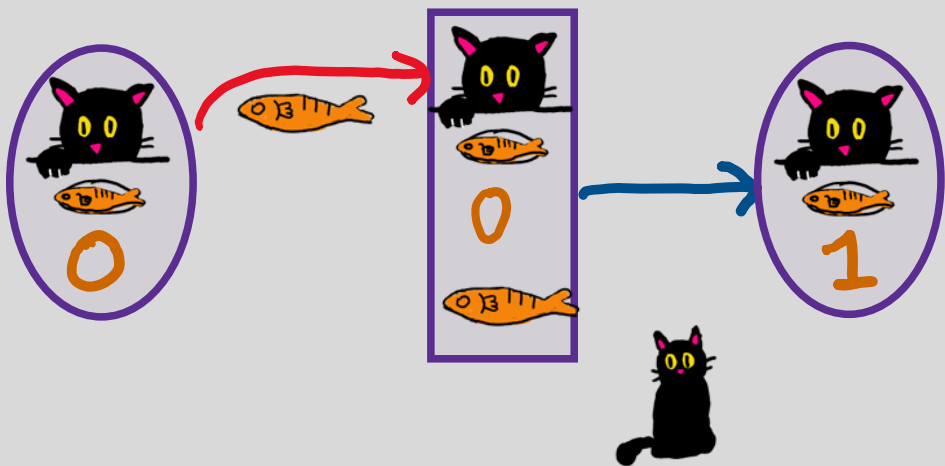


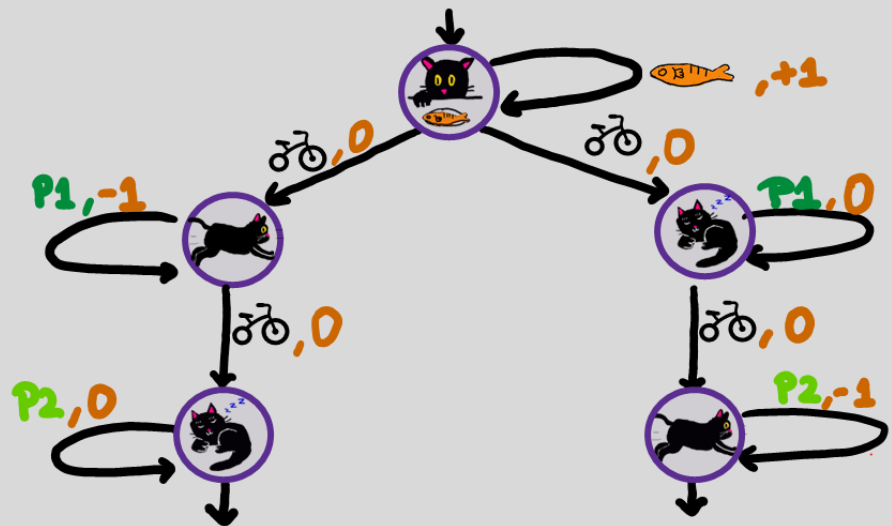
$$L = \{ \text{fish}^i \text{ } \text{bicycle}^j \text{ } \text{bicycle}^k \mid i \geq j \text{ or } i \geq k \}$$



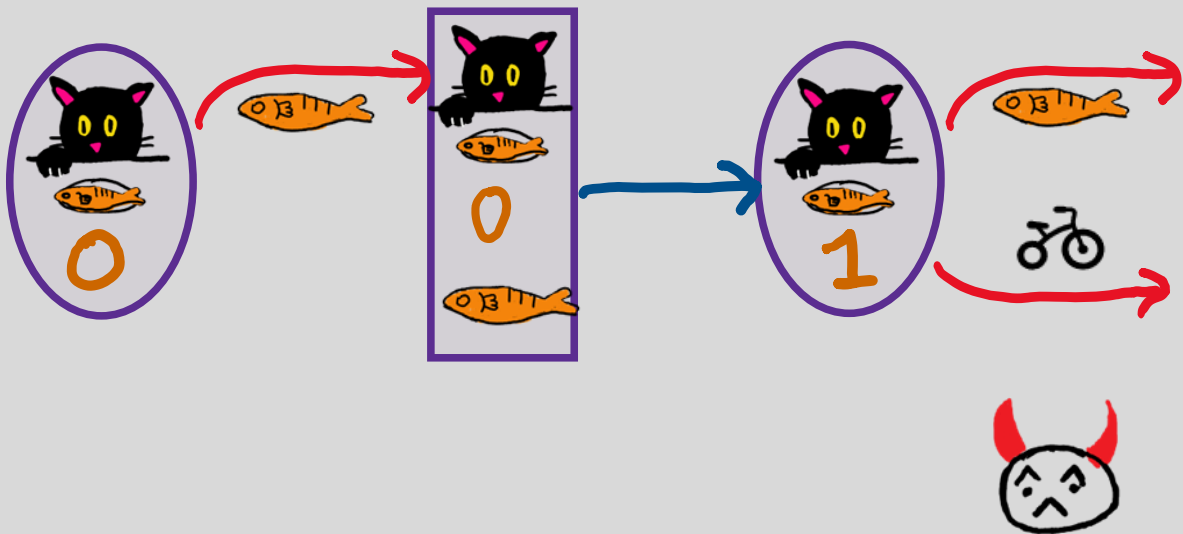


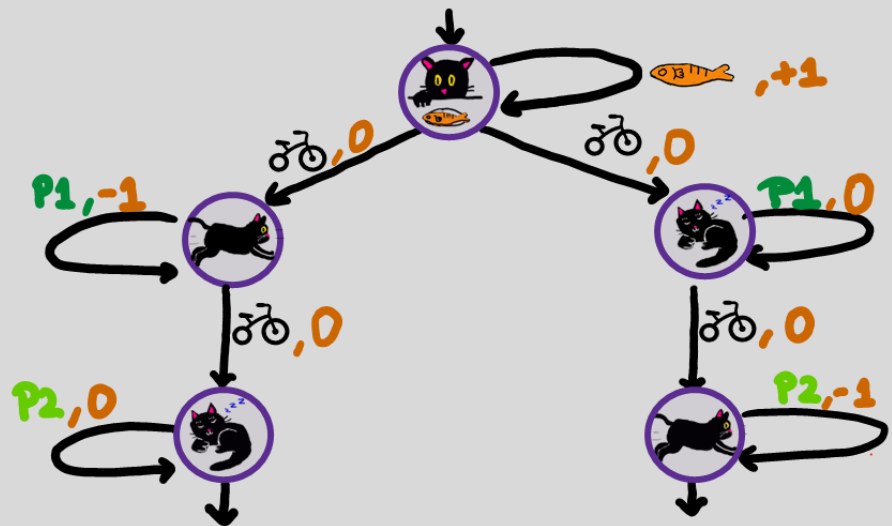
$$L = \{ \text{fish}^i \text{ } \text{bike} \text{ } P1^j \text{ } \text{bike} \text{ } P2^k \mid i \geq j \text{ or } i \geq k \}$$



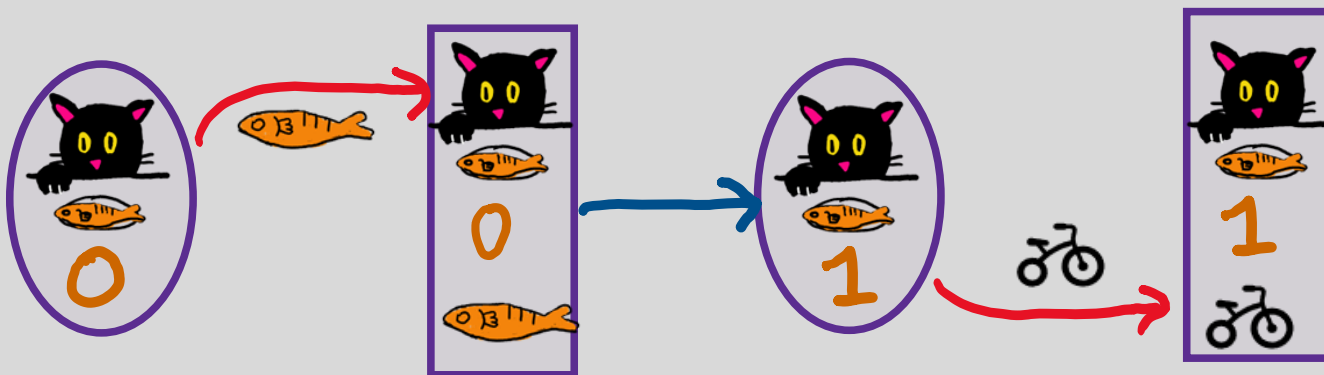


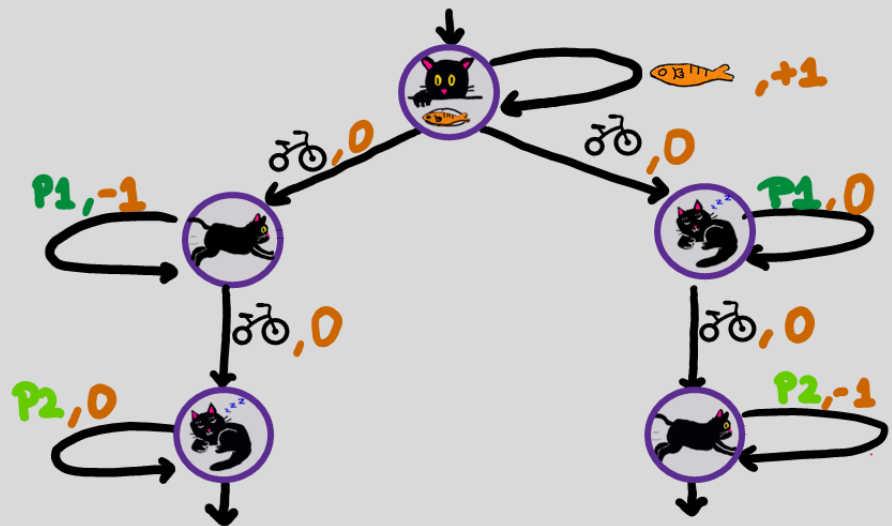
$$L = \{ \text{fish}^i \text{ bicycle } P1^j \text{ bicycle } P2^k \mid i \geq j \text{ or } i \geq k \}$$



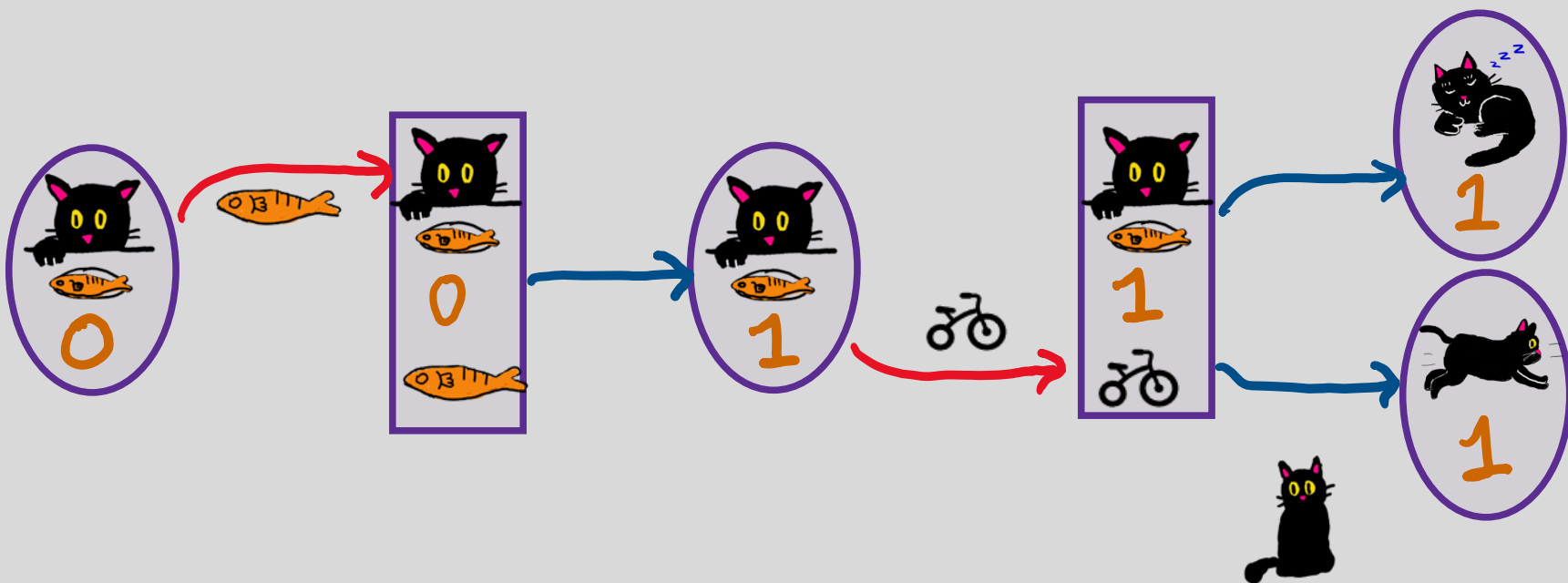


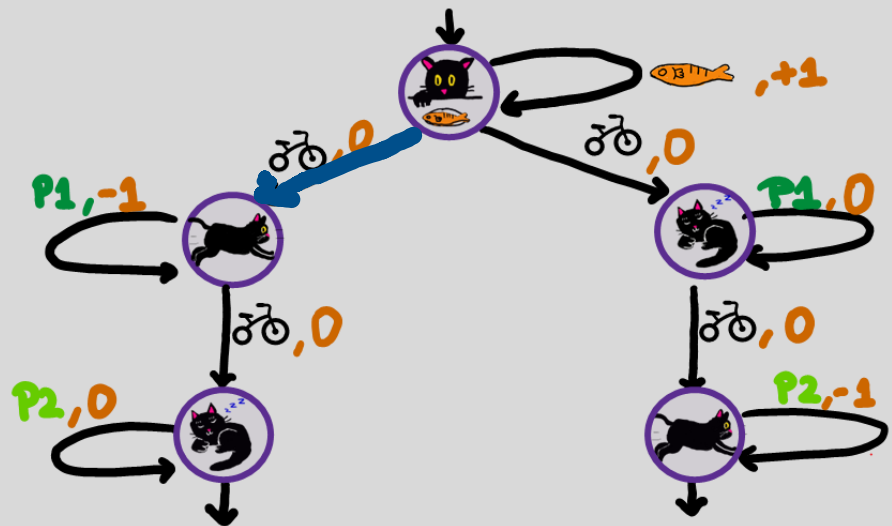
$$L = \{ \text{fish}^i \text{ } \text{bike}^j \text{ } \text{bike}^k \mid i \geq j \text{ or } i \geq k \}$$



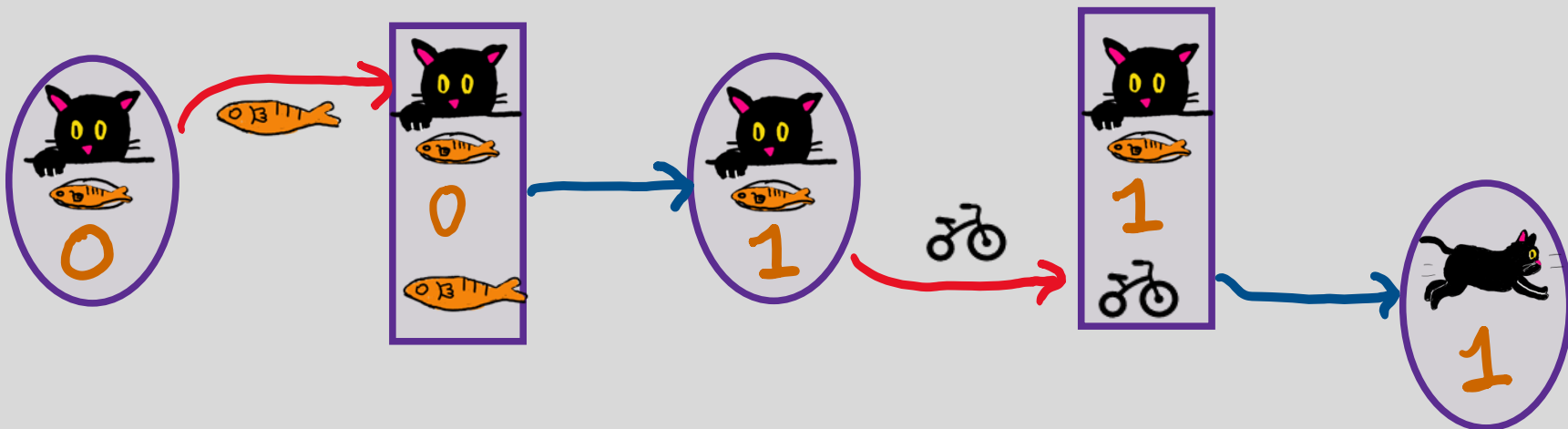


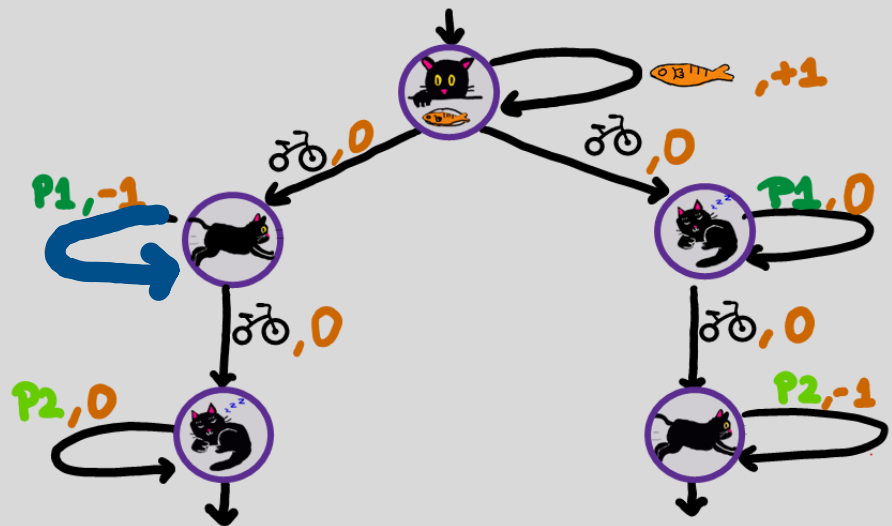
$$L = \{ \text{fish}^i \text{ bike}^{P1^j} \text{ bike}^{P2^k} \mid i \geq j \text{ or } i \geq k \}$$



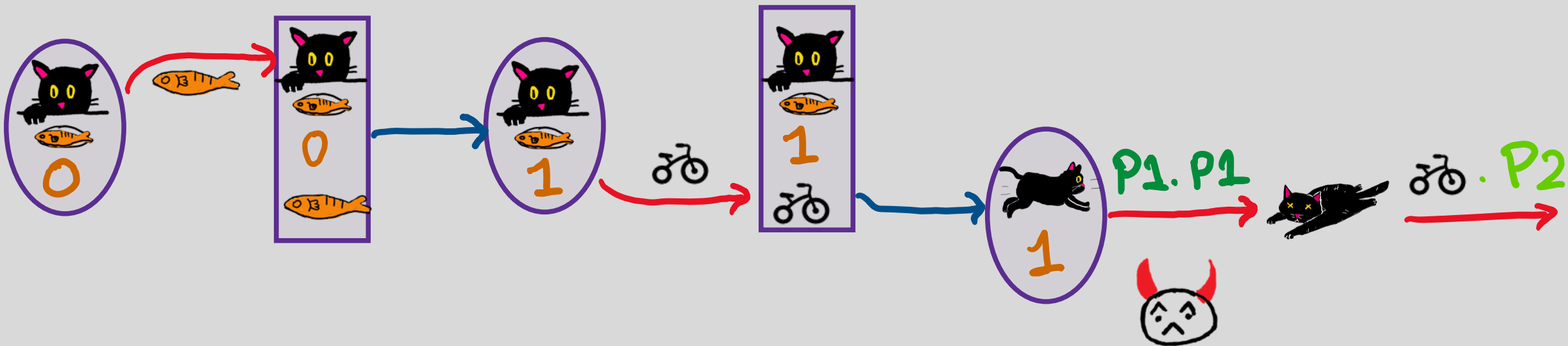


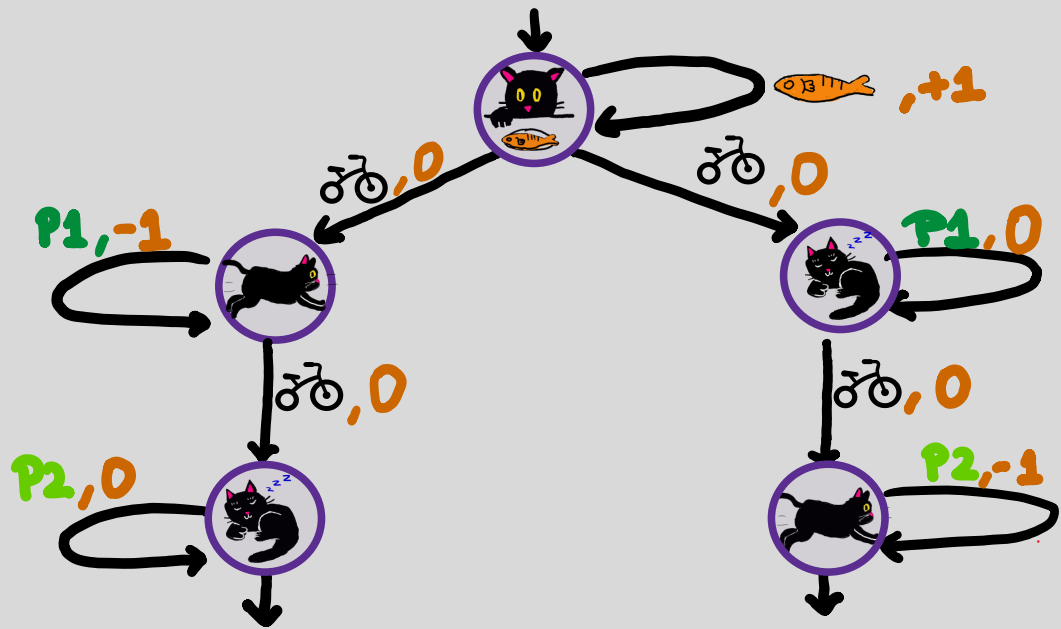
$$L = \{ \text{fish}^i \text{ } \text{bicycle}^{P1^j} \text{ } \text{bicycle}^{P2^k} \mid i \geq j \text{ or } i \geq k \}$$





$$L = \{ \text{fish}^i \text{ bicycle}^j \text{ bicycle}^k \mid i \geq j \text{ or } i \geq k \}$$





$$L = \{ \text{fish}^i \text{ bicycle } P1^j \text{ bicycle } P2^k \mid i \geq j \text{ or } i \geq k \}$$



P1

P1



P2

∈ L



0



1



1



0



×

Definition: A one-counter net N is HD iff

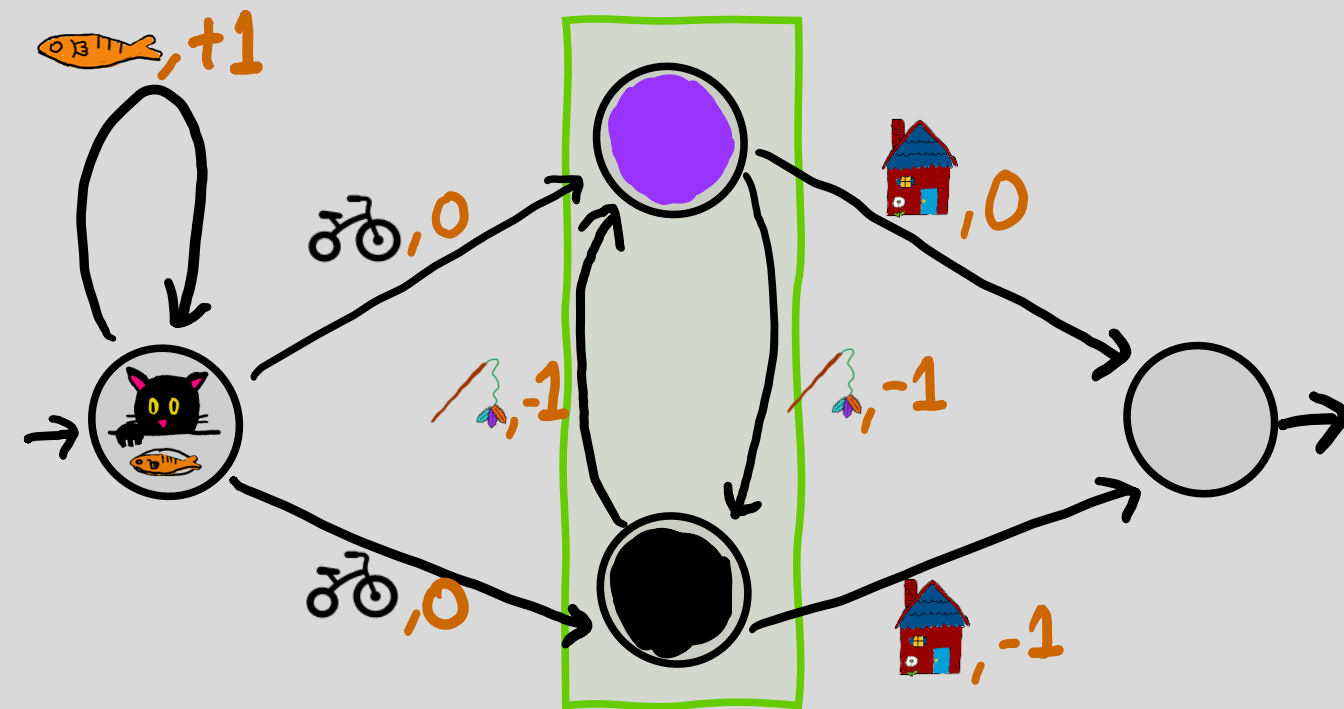


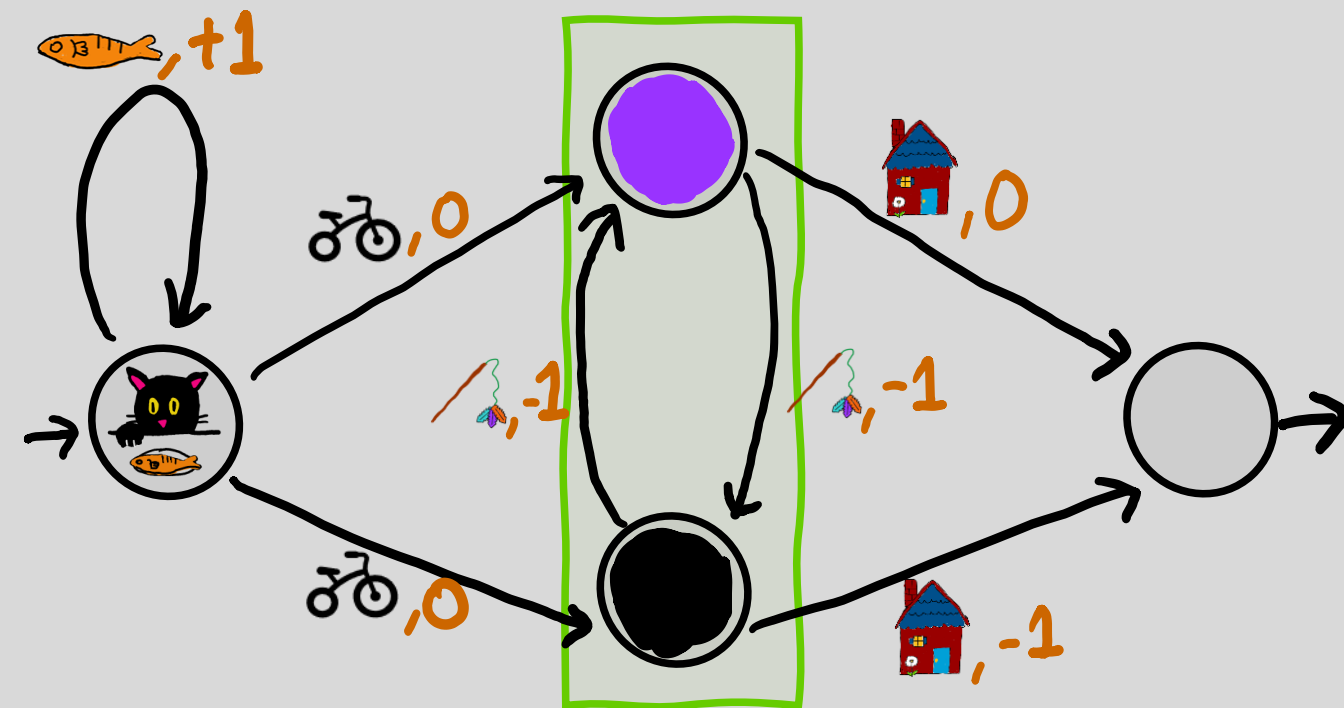
wins

HD-game.

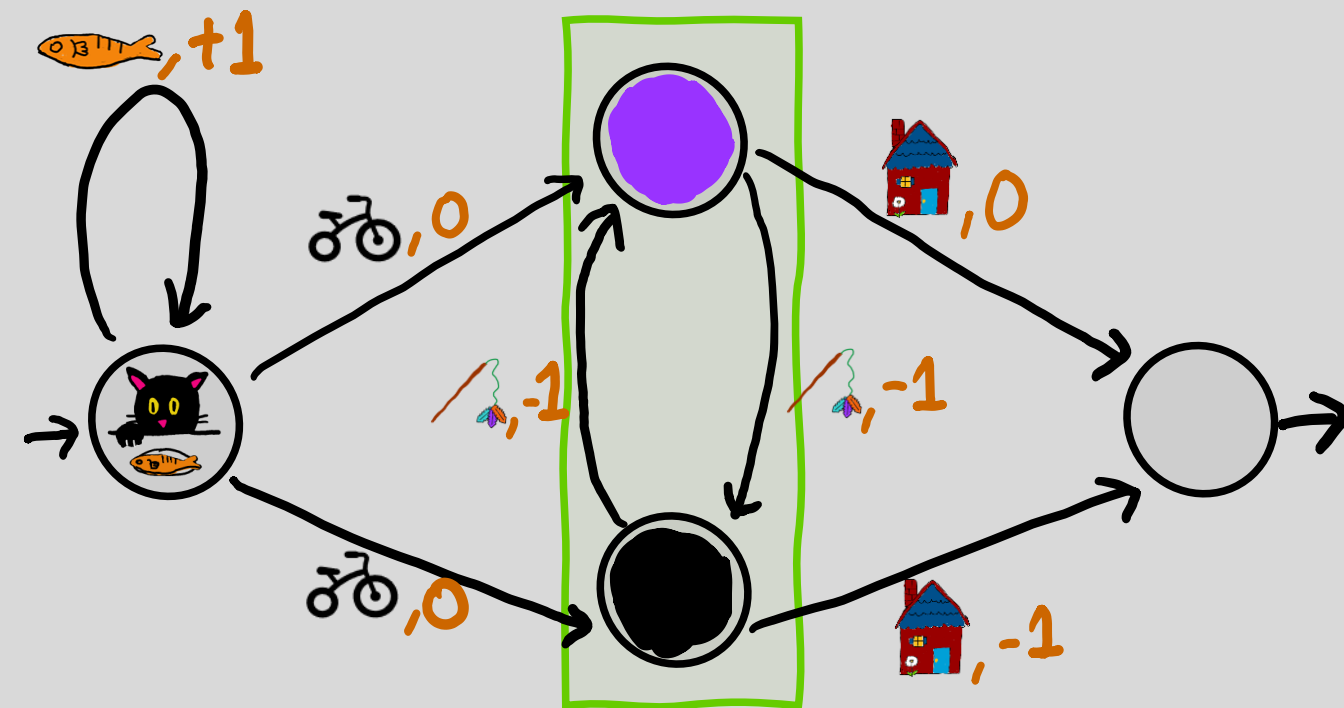
Resolver: A winning strategy of  .

2. Resolvers



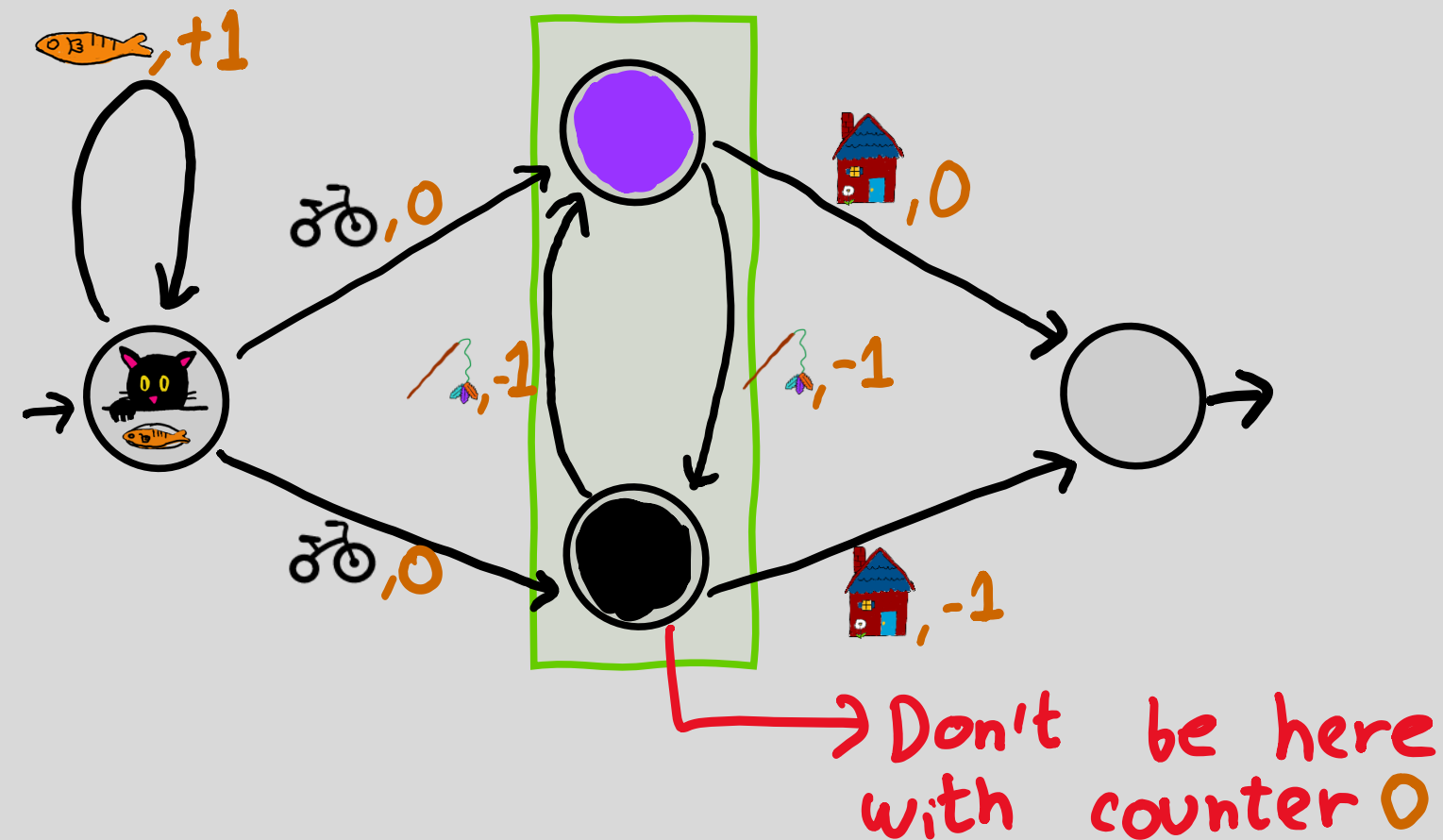


$$L = \{ \text{fish}^n \text{ bicycle}^m \text{ fishing rod}^m \text{ house} \mid n \geq m \}$$



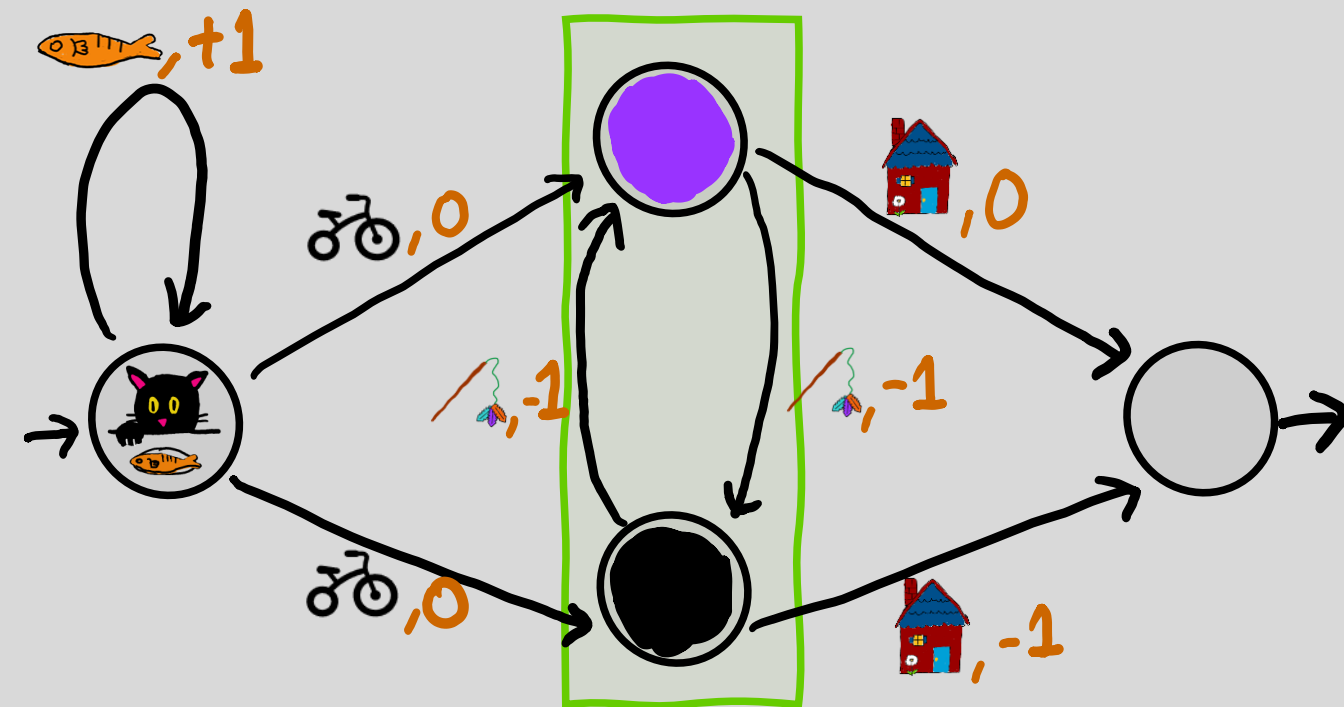
At , on  :

$$L = \{ \text{fish}^n \text{ bicycle}^m \text{ fishing rod}^m \text{ house} \mid n \geq m \}$$



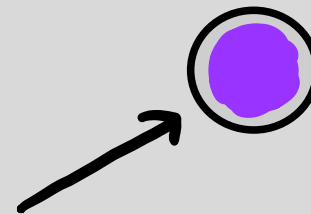
At , on  :

$$L = \{ \text{fish}^n \text{ bicycle}^m \text{ house} \mid n \geq m \}$$



At , on  :

Counter = $2n$:



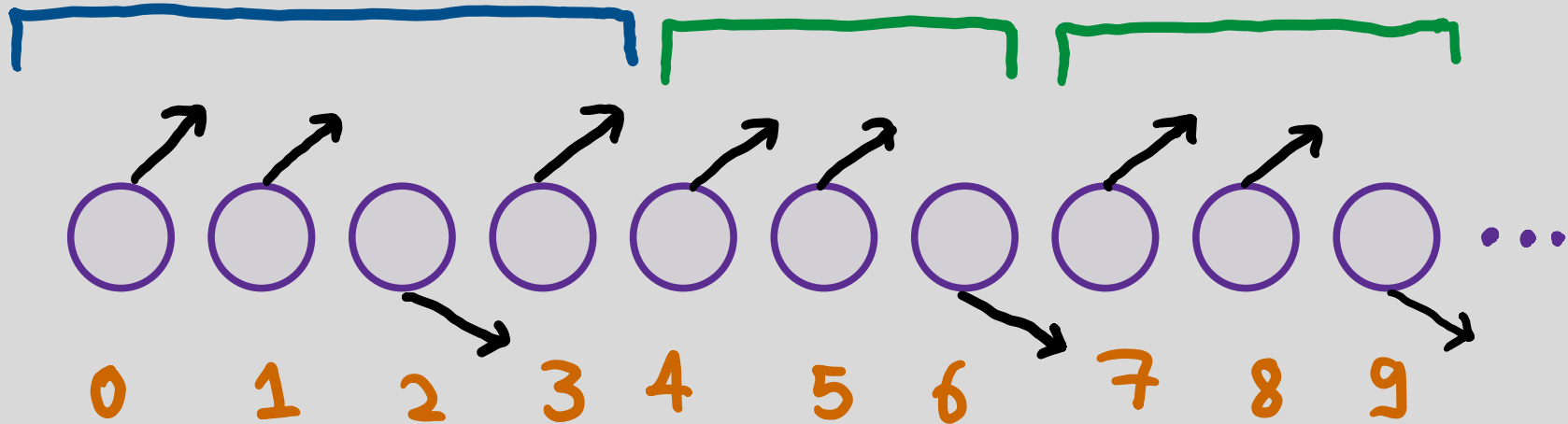
Counter = $2n + 1$:



$$L = \{ \text{fish}^n \text{ bicycle}^m \text{ fishing rod}^m \text{ house} \mid n \geq m \}$$

Theorem: Given an history-deterministic OCN,
there is a semilinear strategy for  that
is effectively computable.

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is effectively computable.

Proof:



Token Game:



Token Game:



1.  selects letter

2.  selects transition on 

3.  selects transition on 



Token Game:

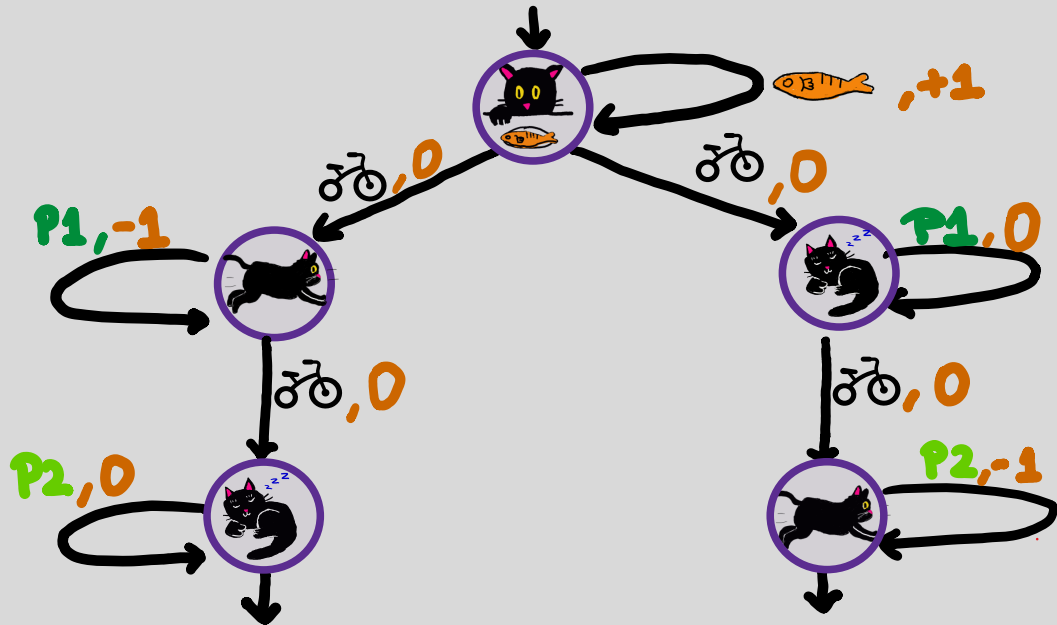


1.  selects letter

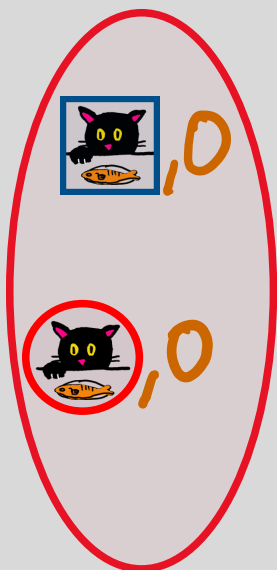
2.  selects transition on 

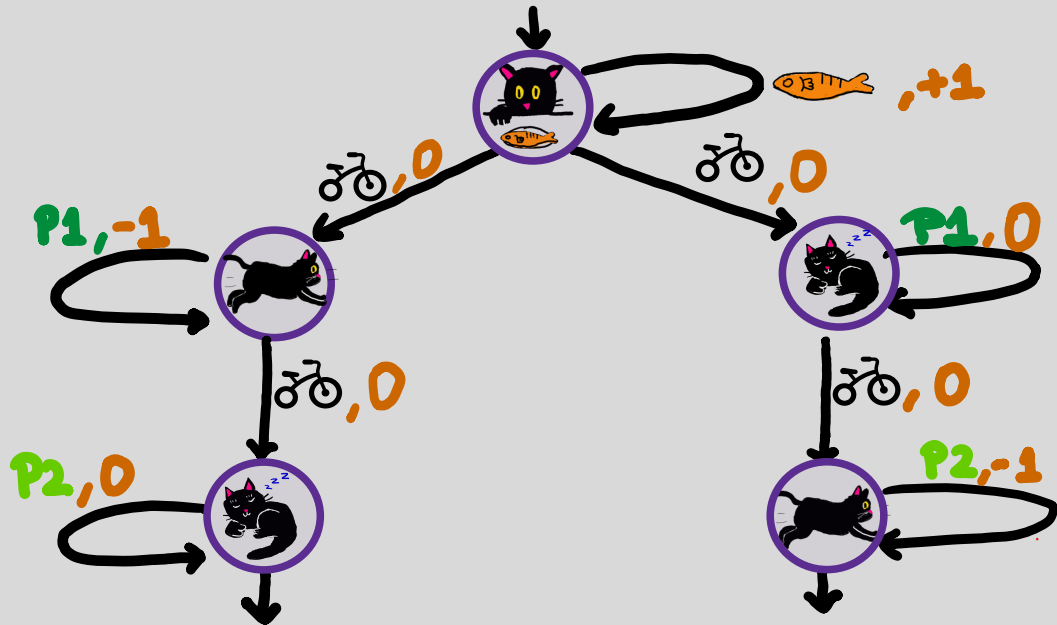
3.  selects transition on 

Winning condition of : If 's run on  is accepting and 's run on  is rejecting.



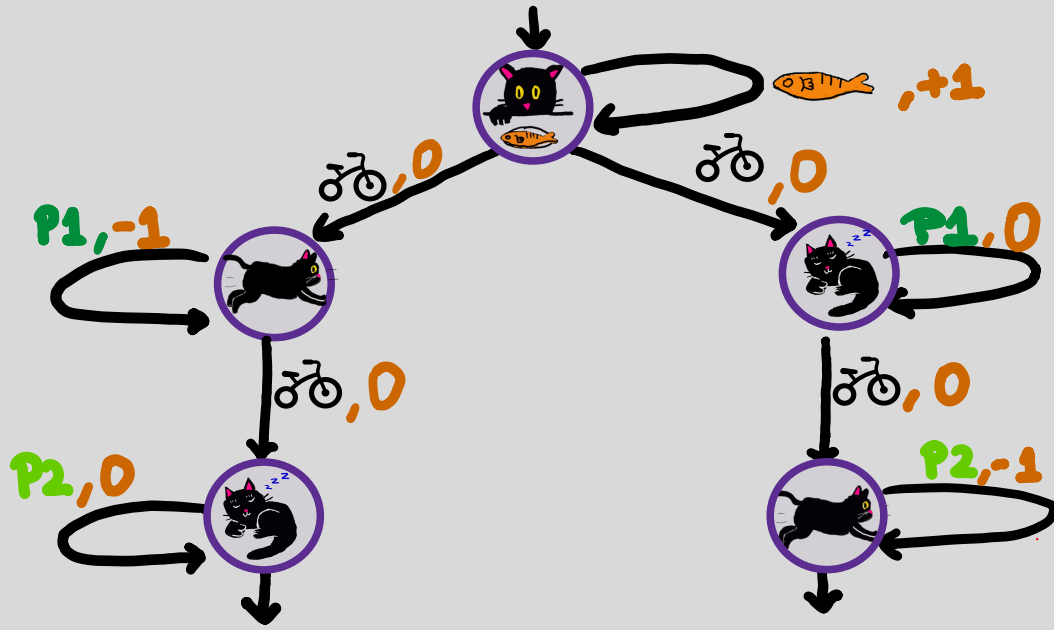
$$L = \{ \text{fish}^i \text{ } \text{bicycle} \text{ } P1^j \text{ } \text{bicycle} \text{ } P2^k \mid i \geq j \text{ or } i \geq k \}$$



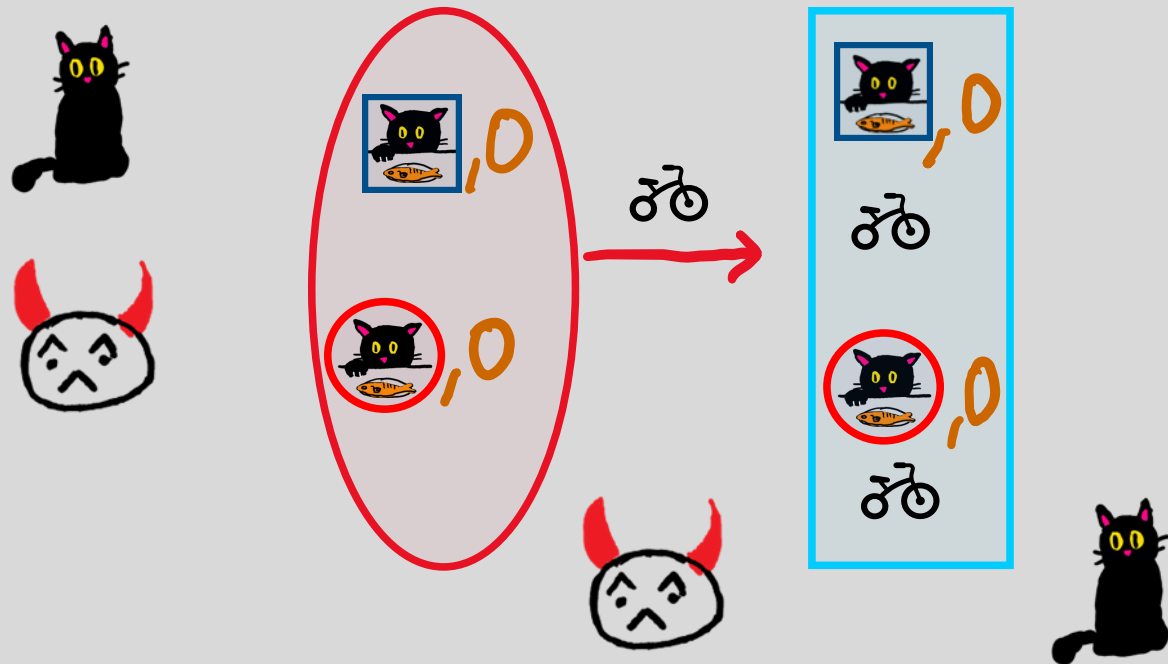


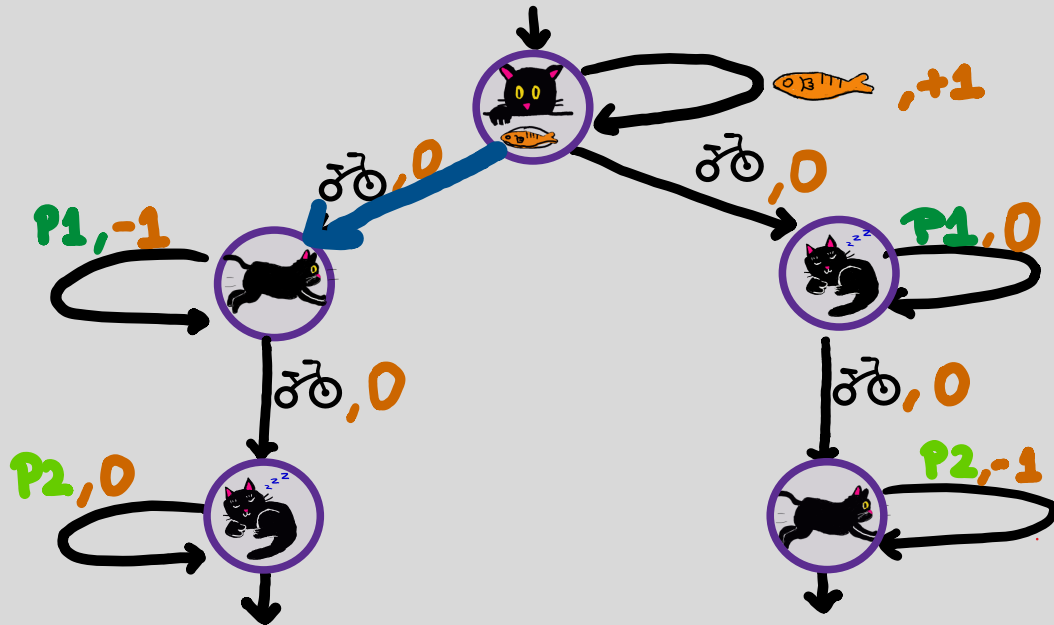
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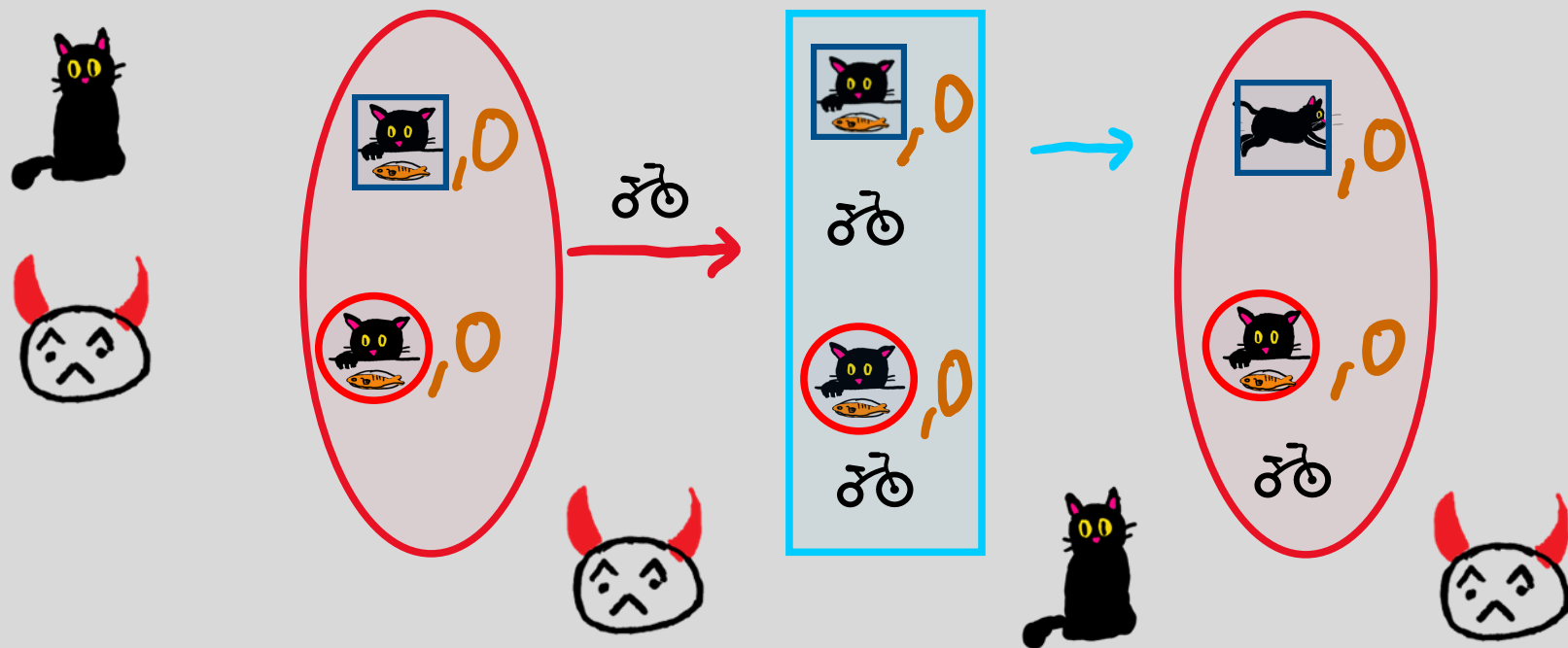


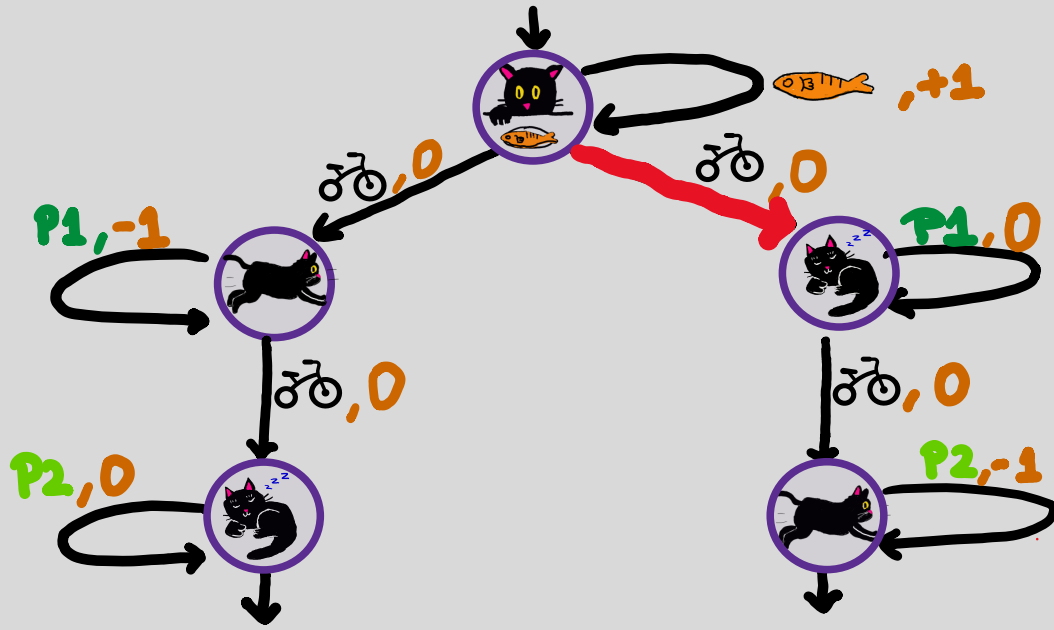
$$L = \{ \text{fish}^i \rightarrow P1^j \rightarrow P2^k \mid i \geq j \text{ or } i \geq k \}$$



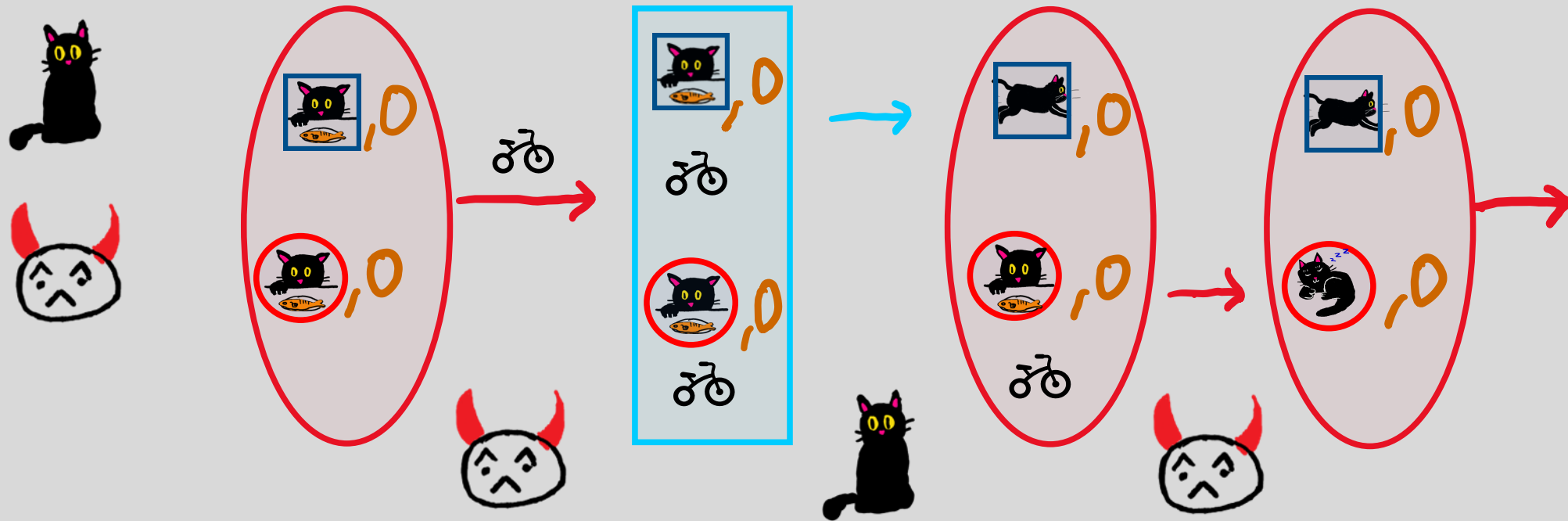


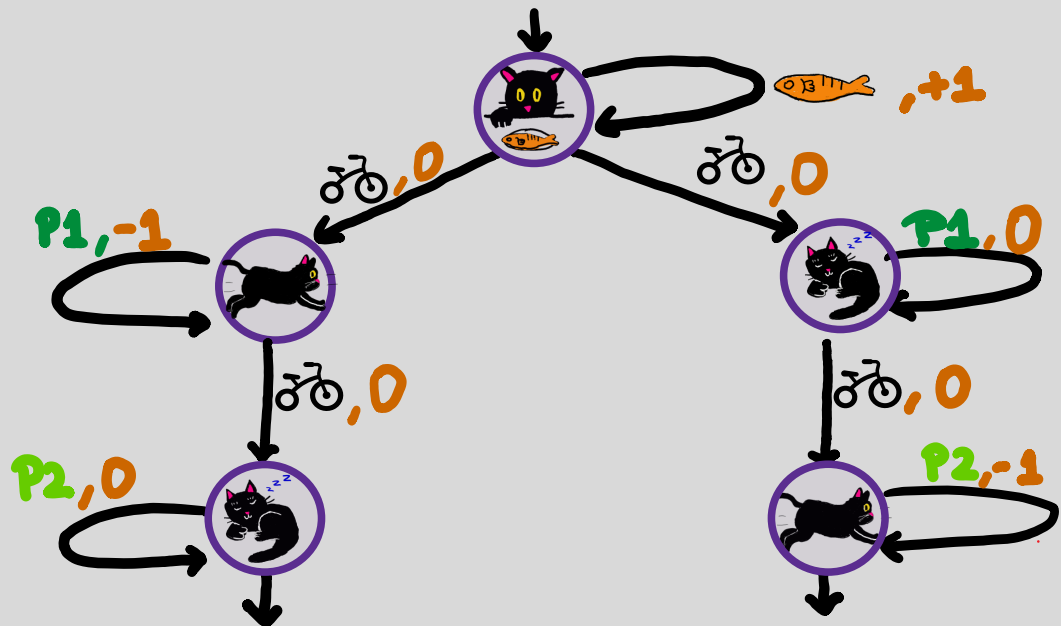
$$L = \{ \text{fish}^i \rightarrow P1^j \rightarrow P2^k \mid i \geq j \text{ or } i \geq k \}$$



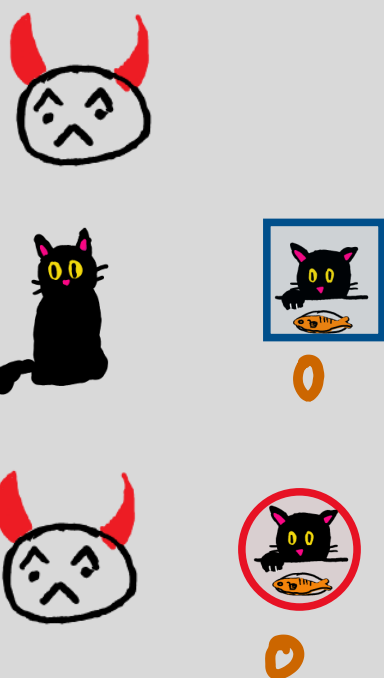


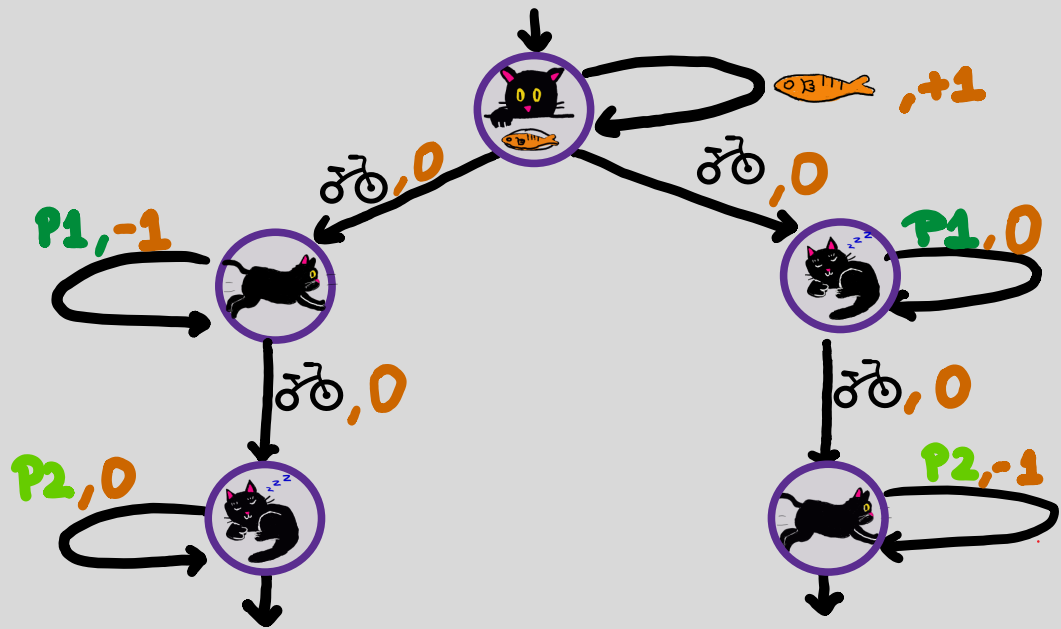
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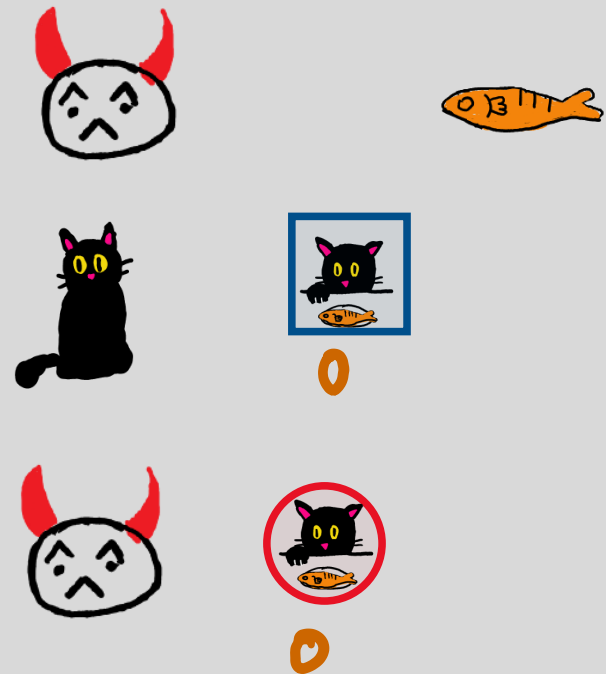


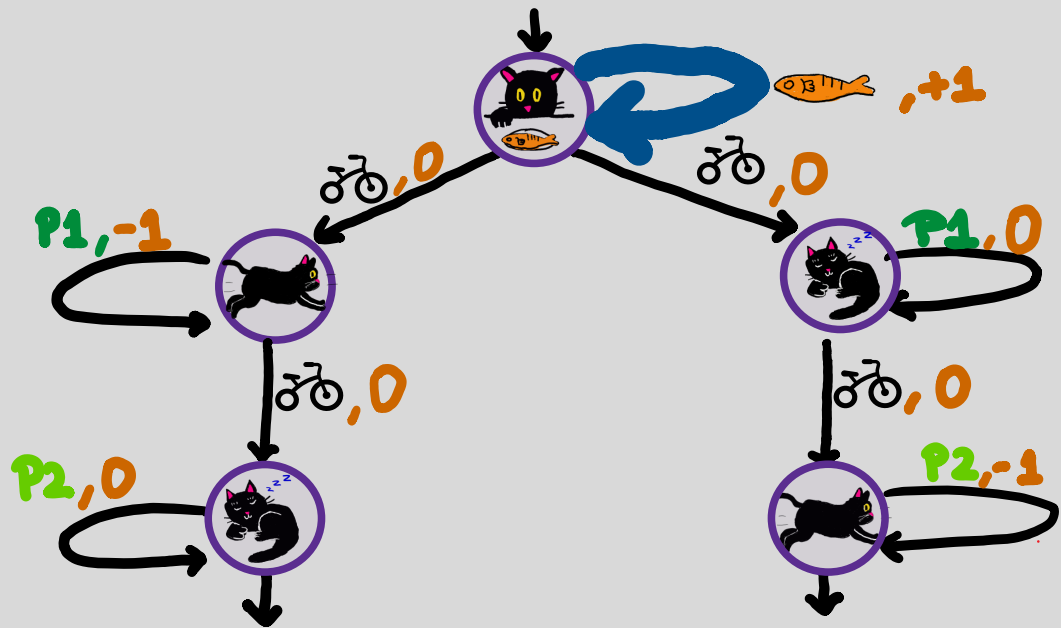
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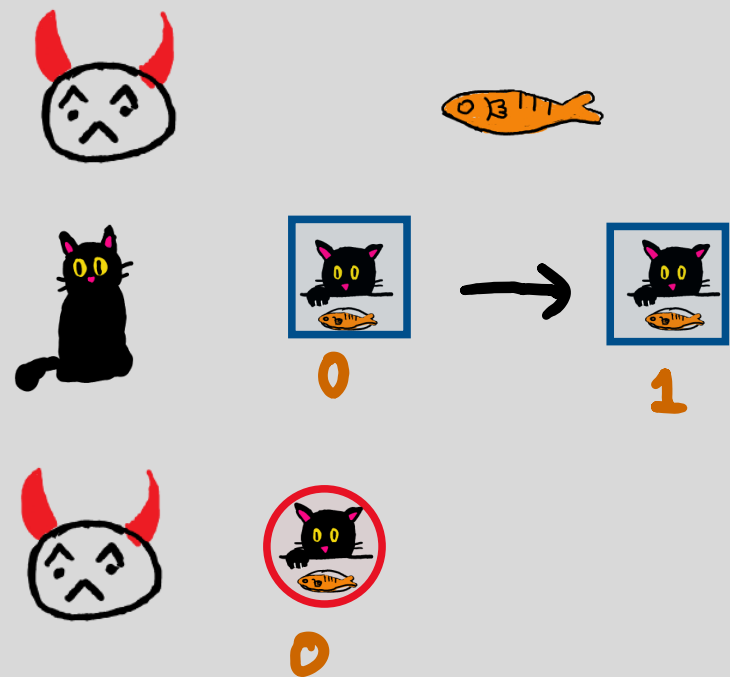


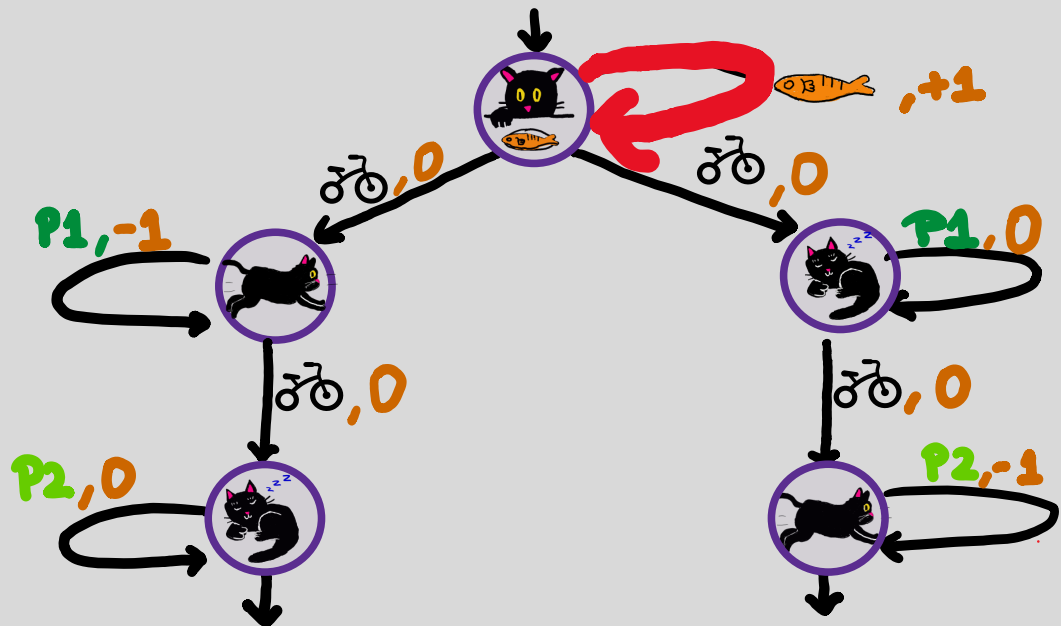
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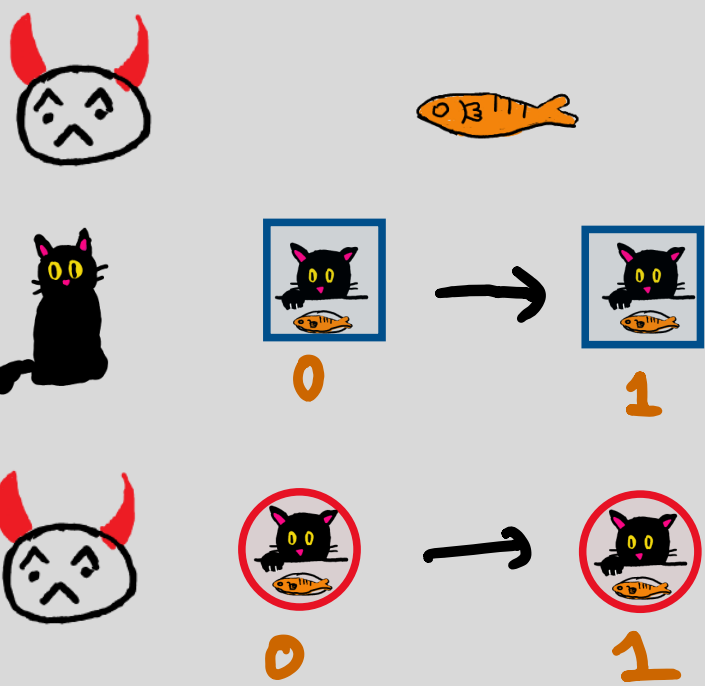


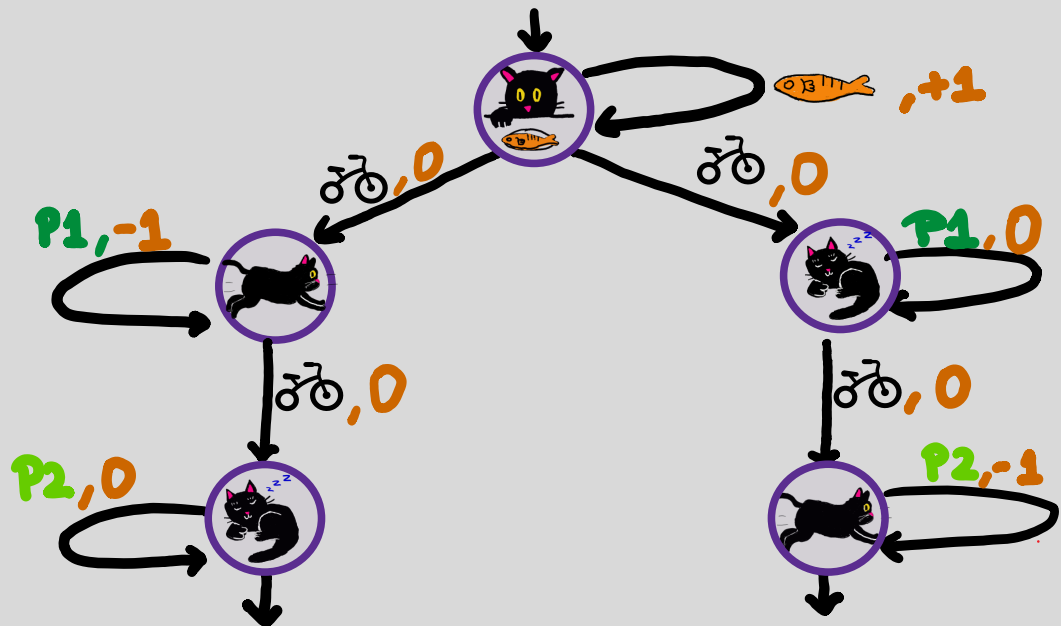
$$L = \{ \text{fish}^i \text{ } \text{bicycle}^j \text{ } \text{P1}^j \text{ } \text{P2}^k \mid i \geq j \text{ or } i \geq k \}$$



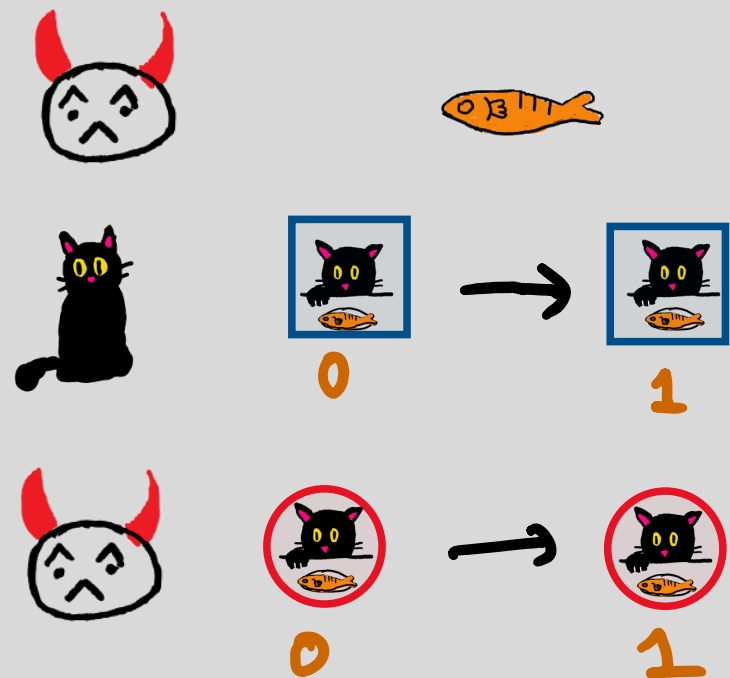


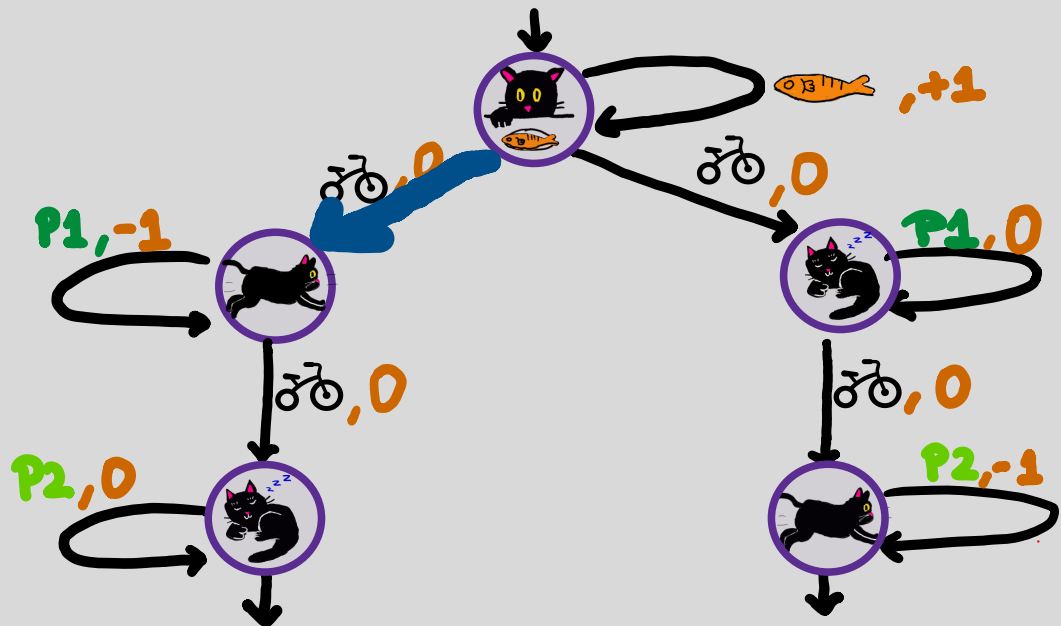
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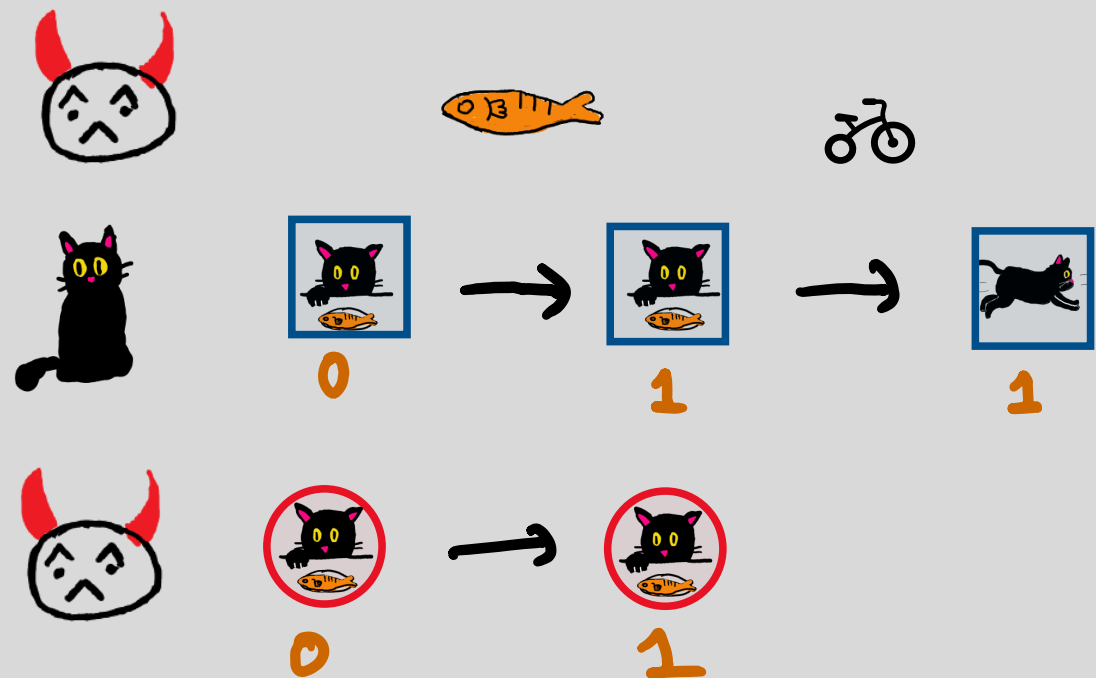


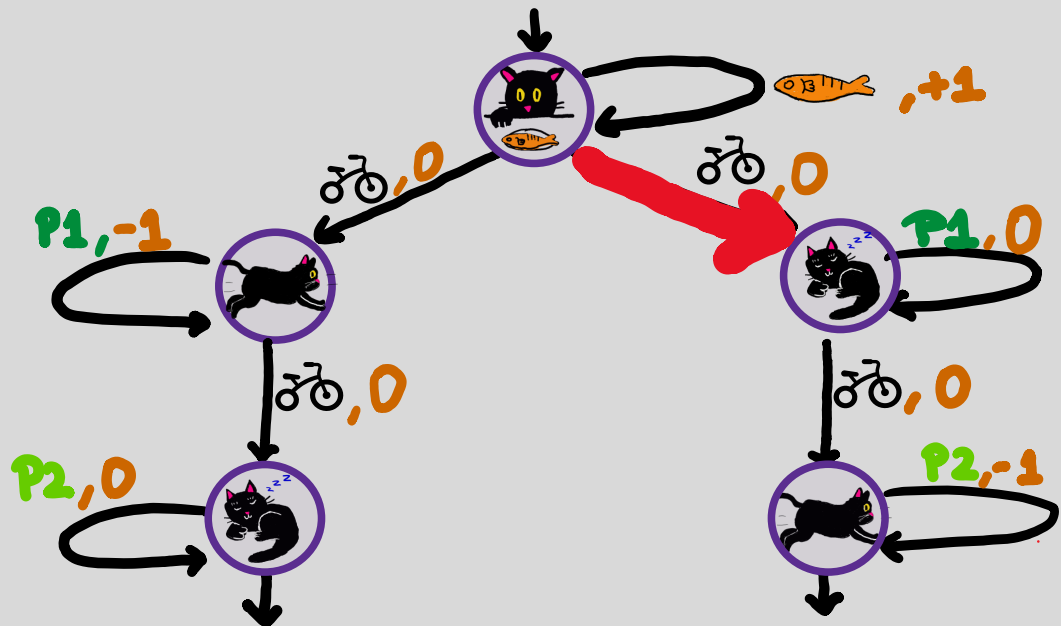
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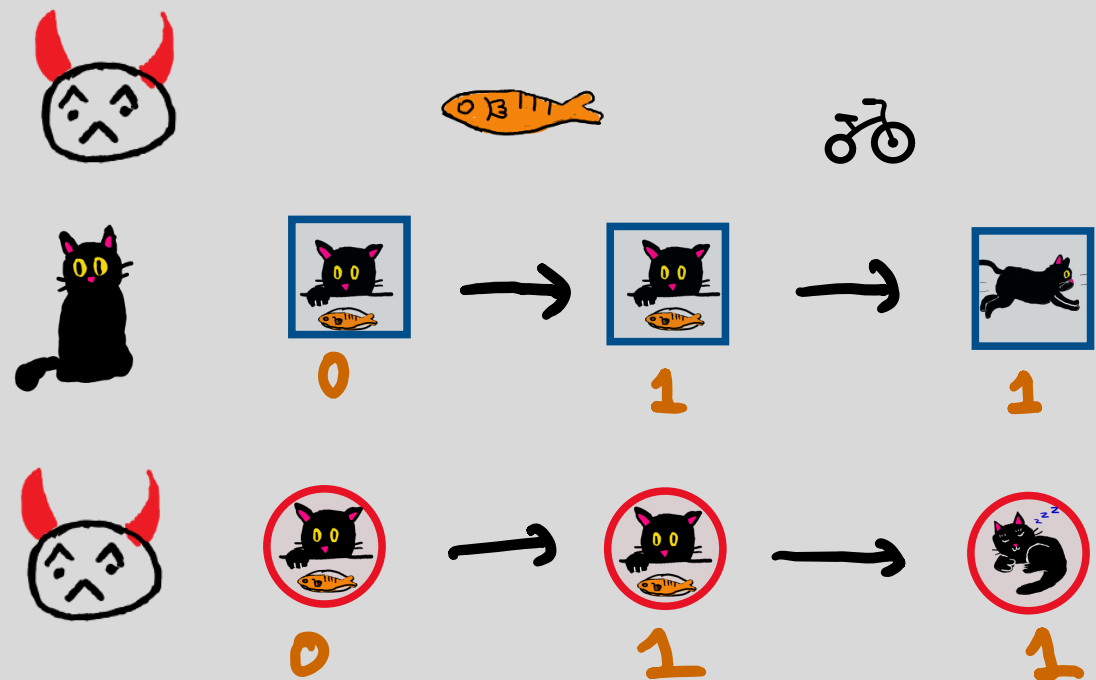


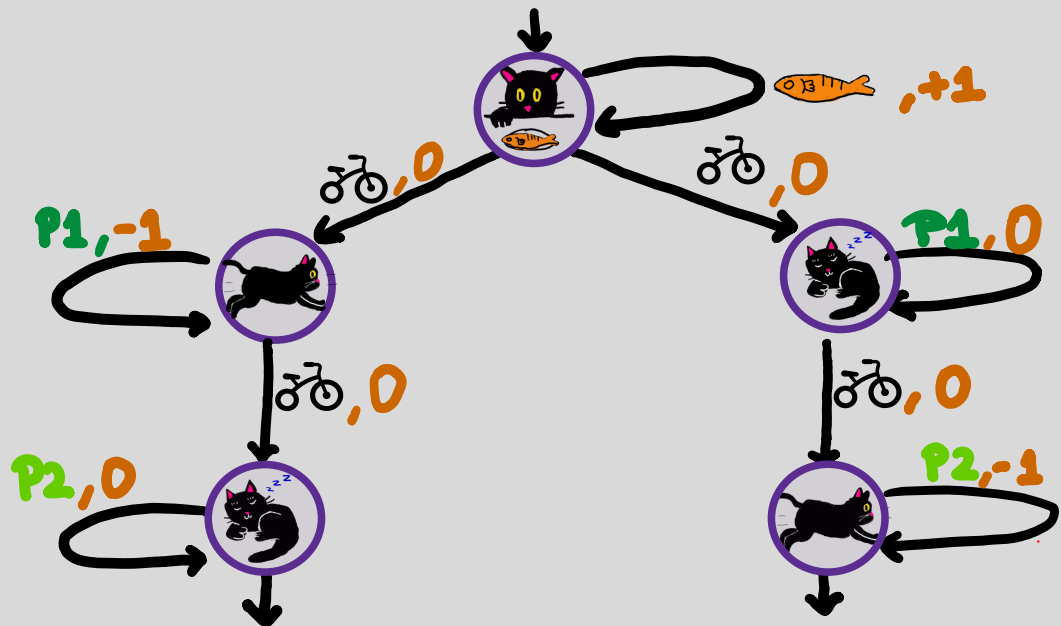
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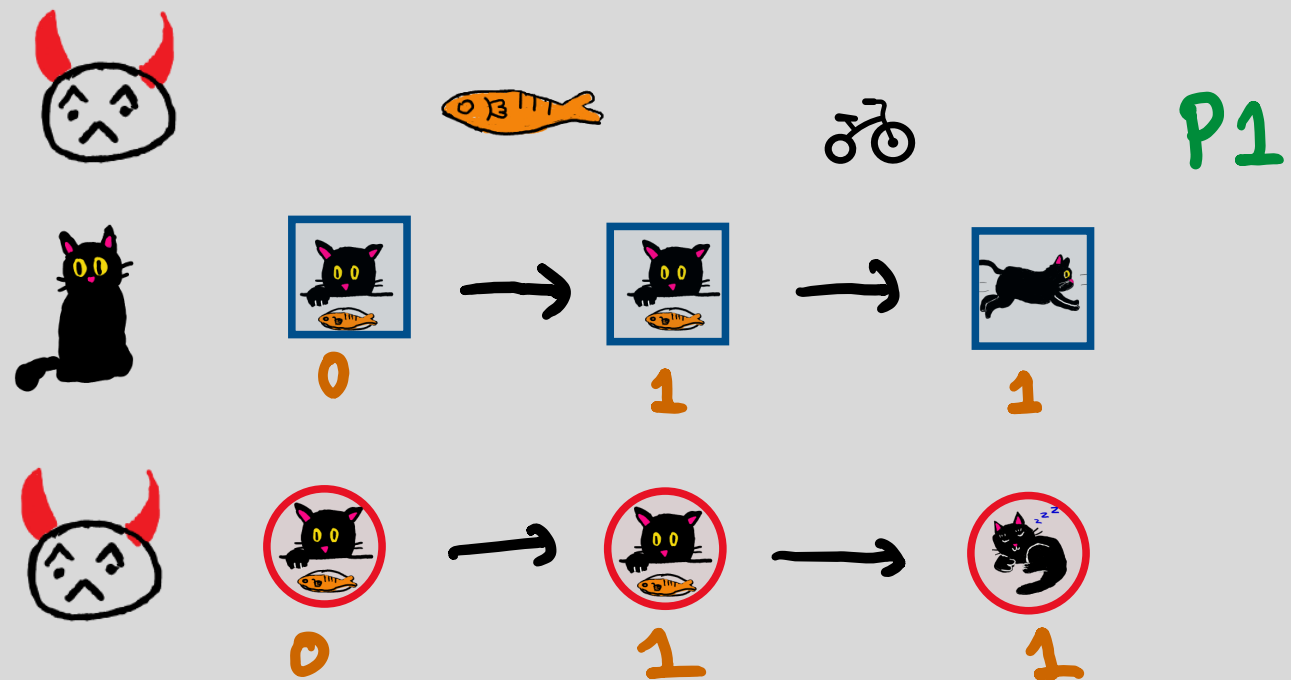


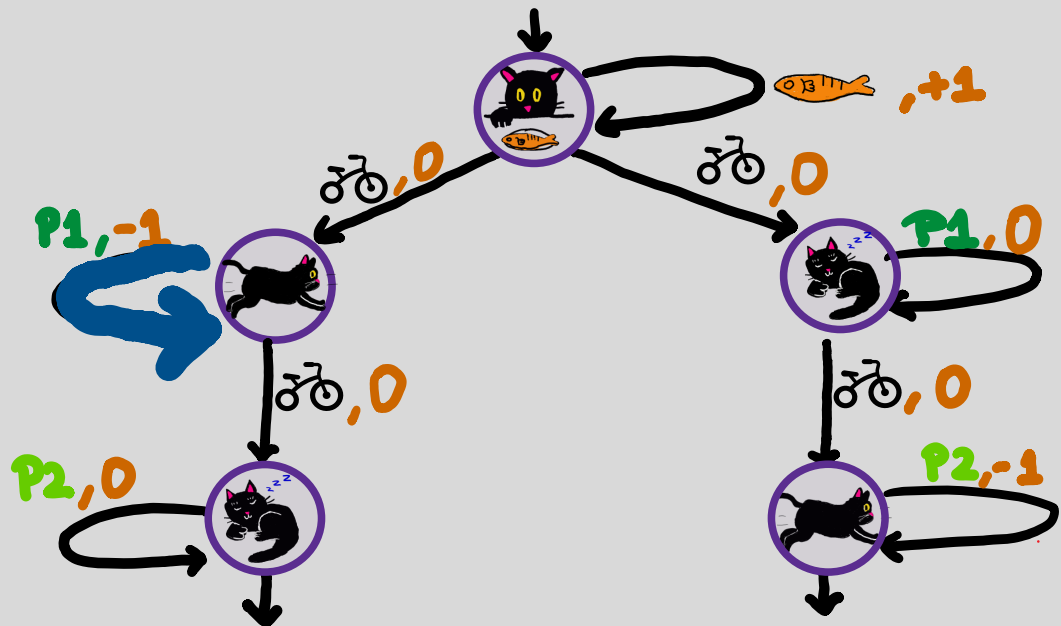
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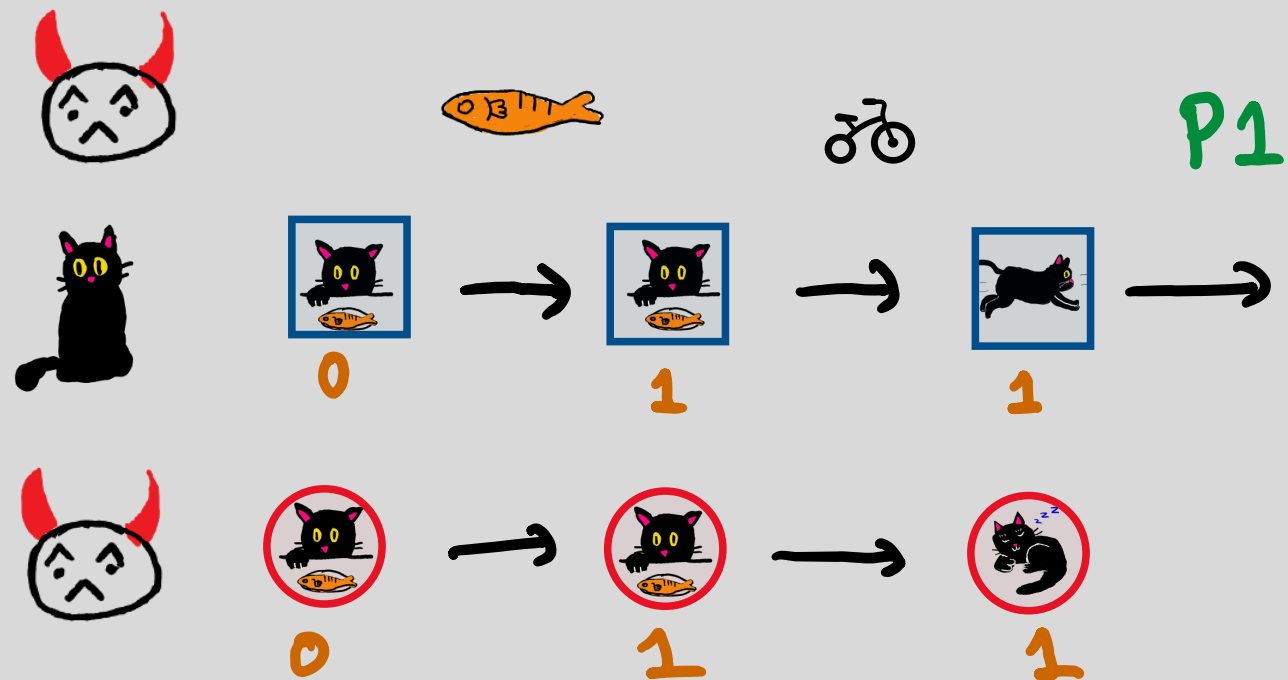


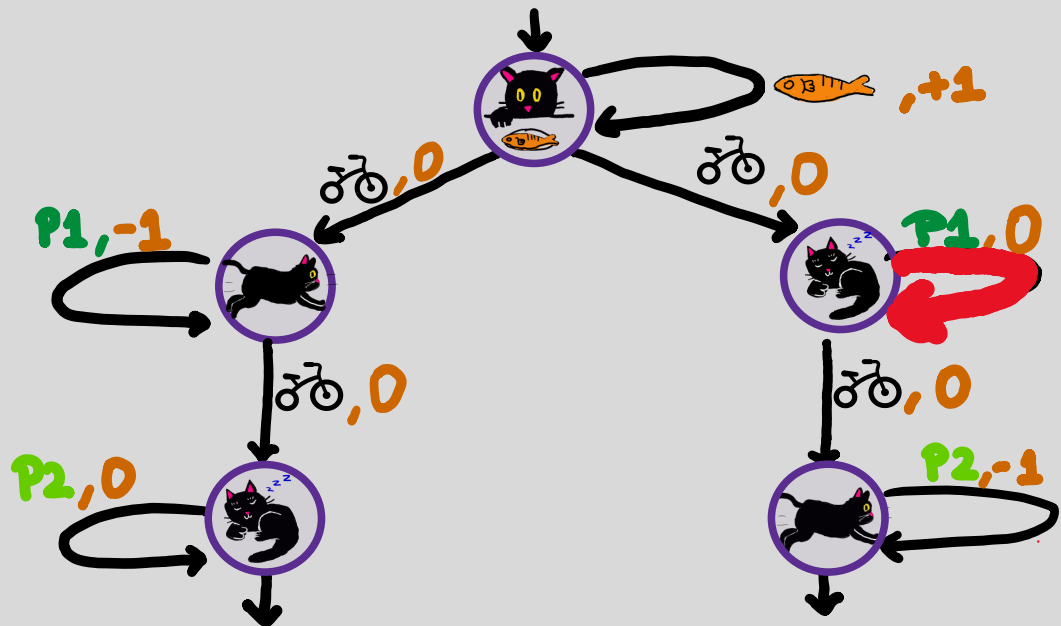
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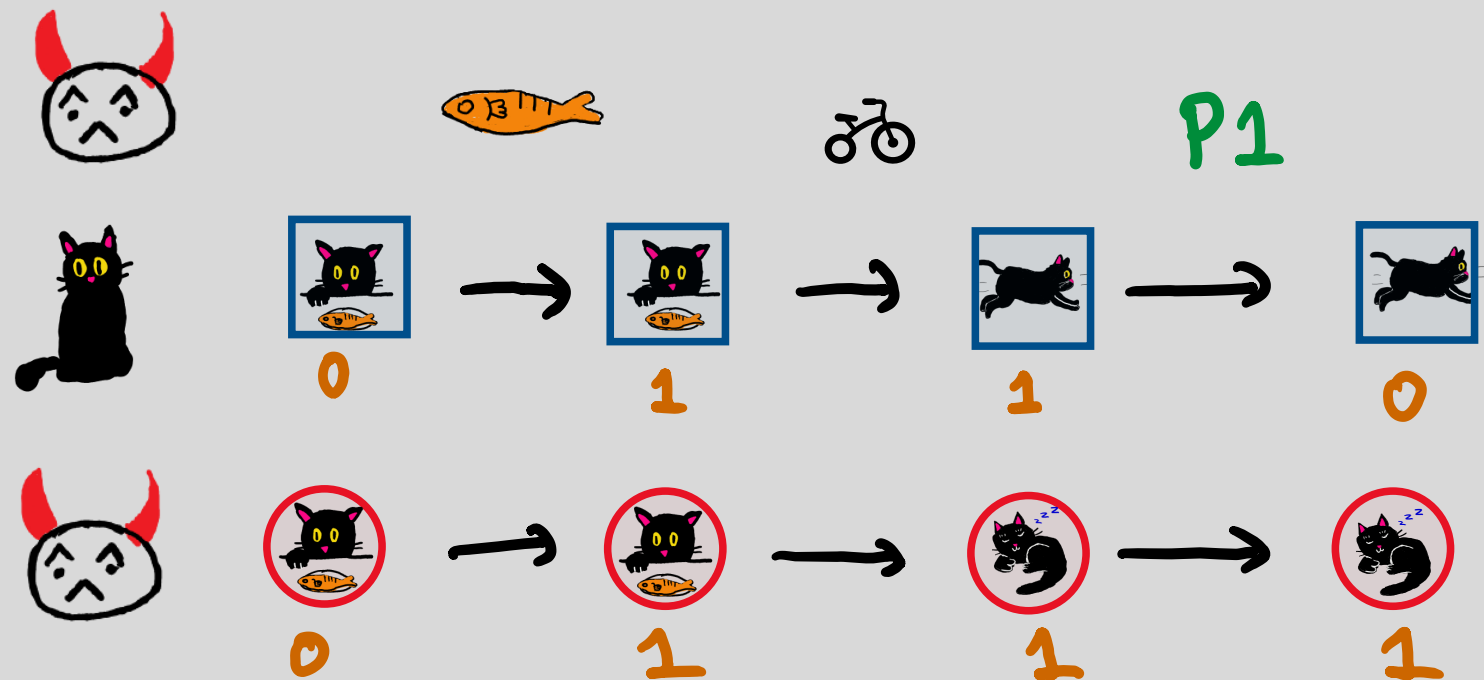


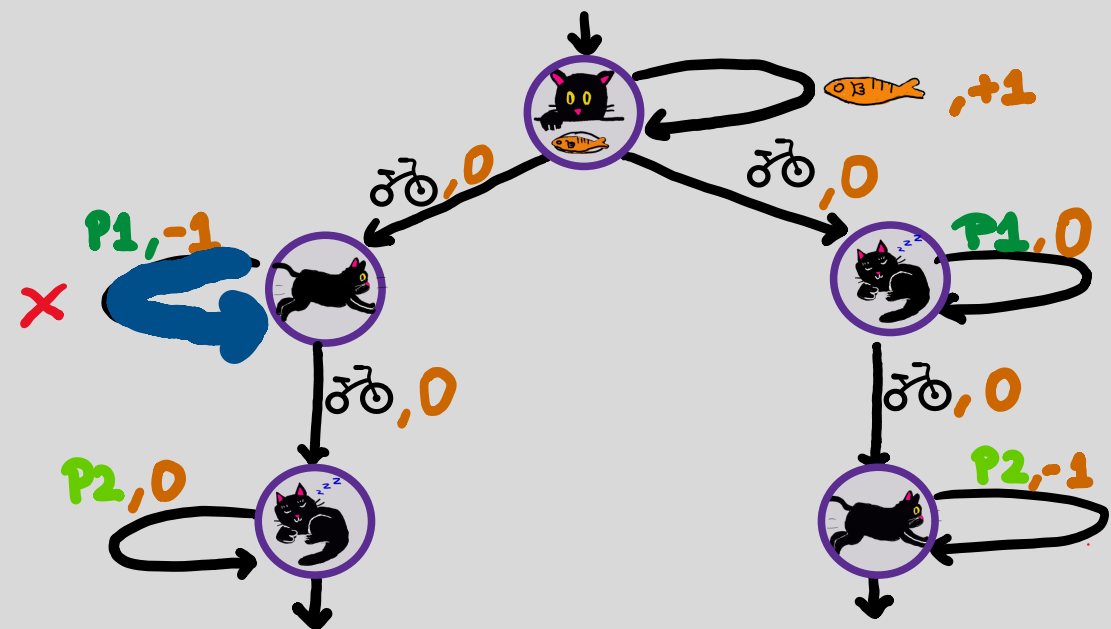
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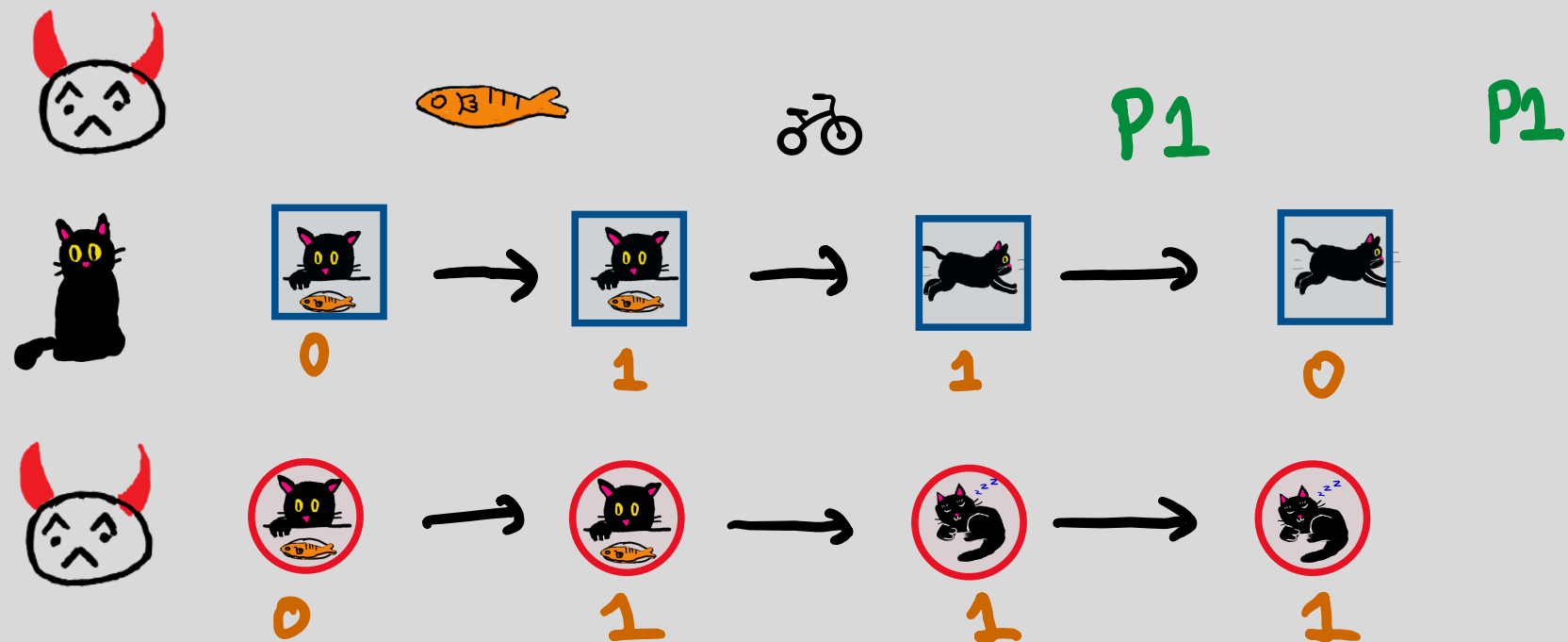


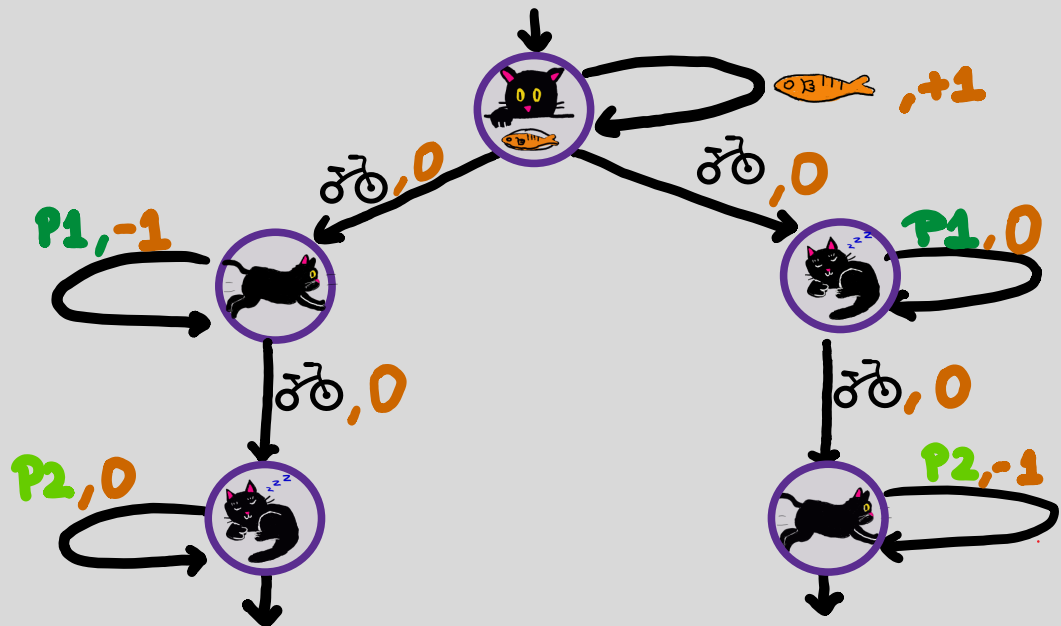
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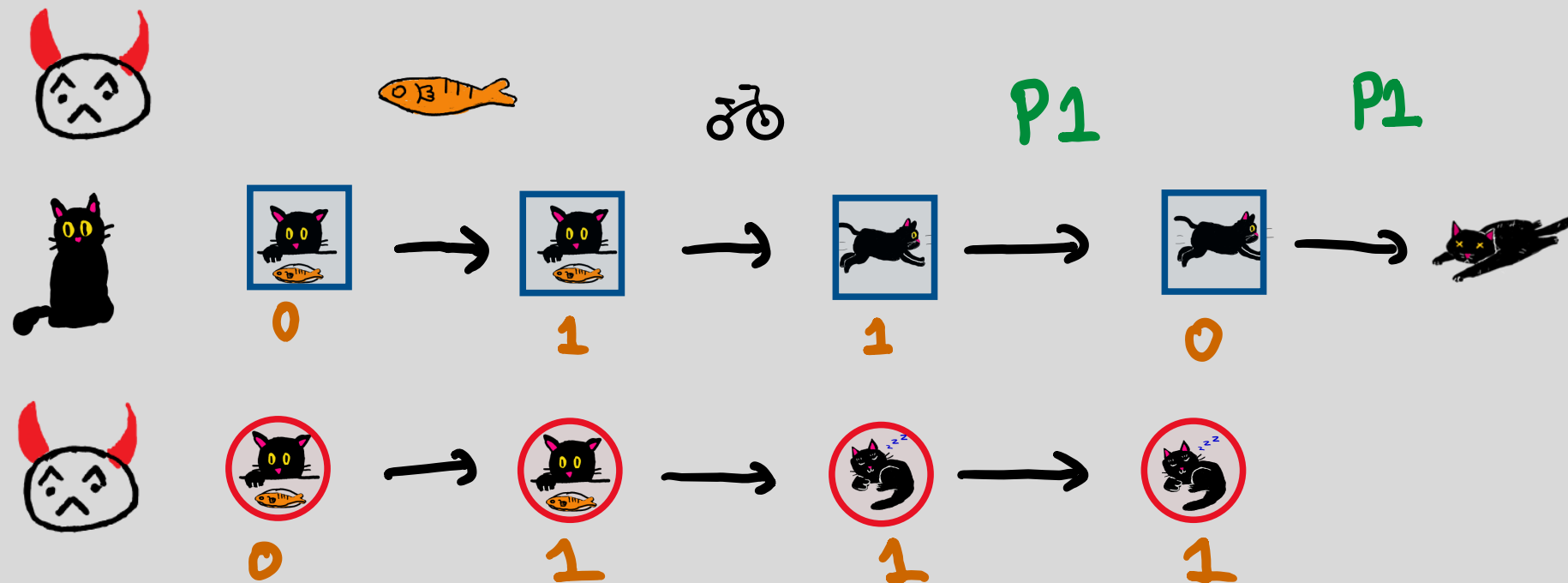


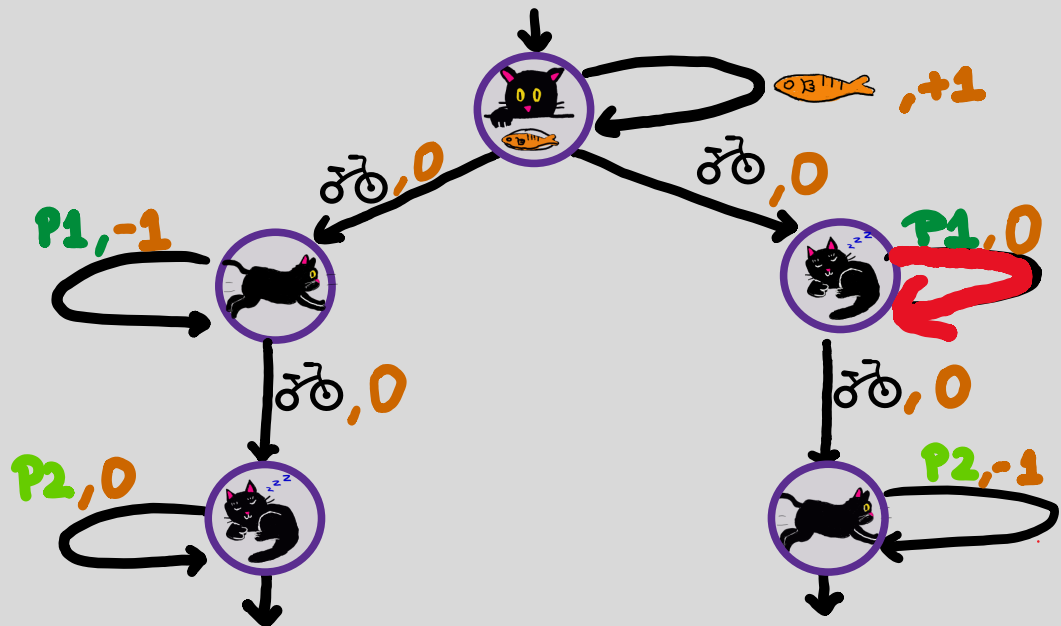
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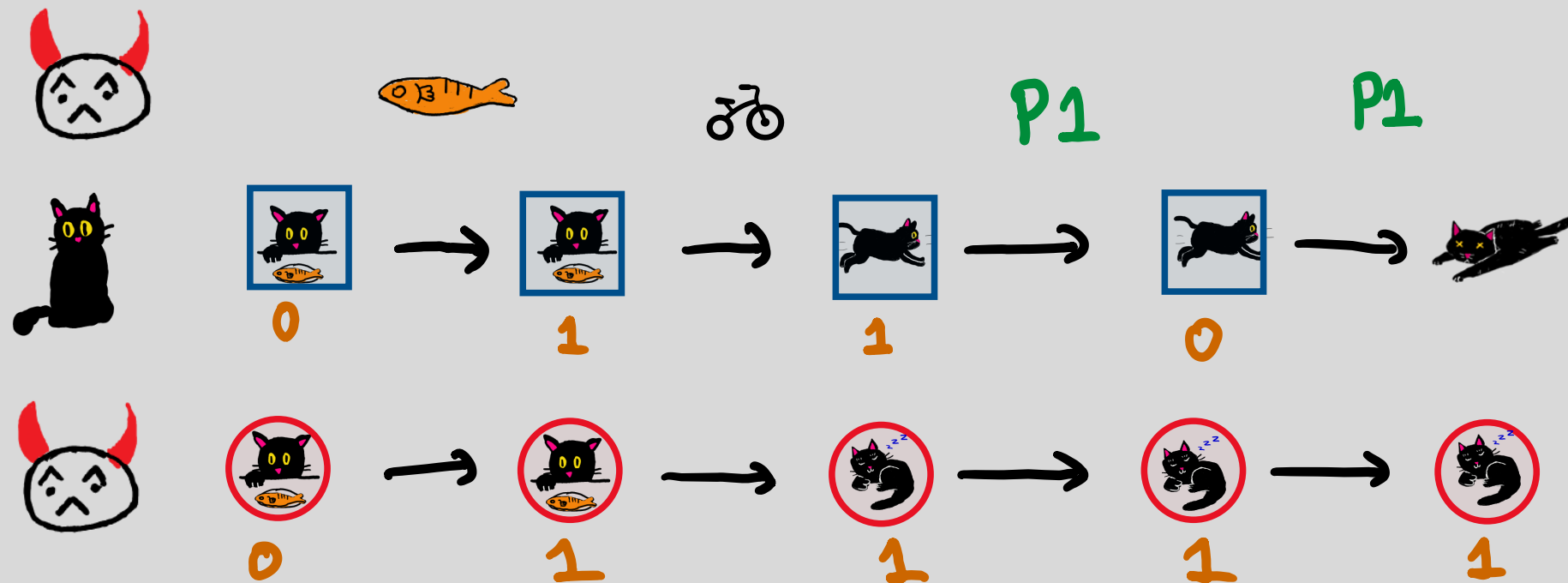


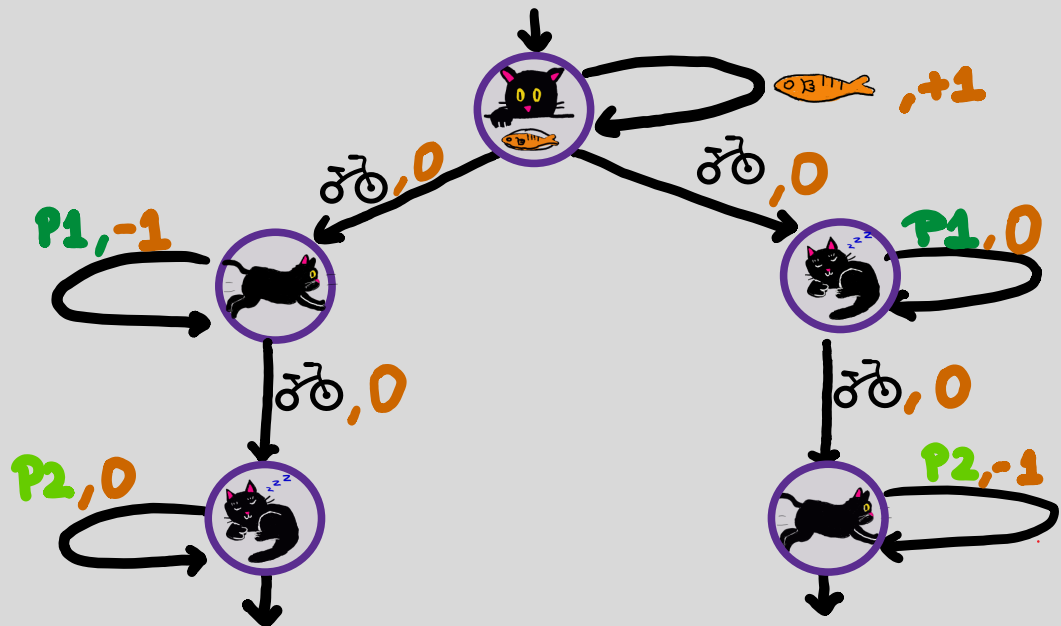
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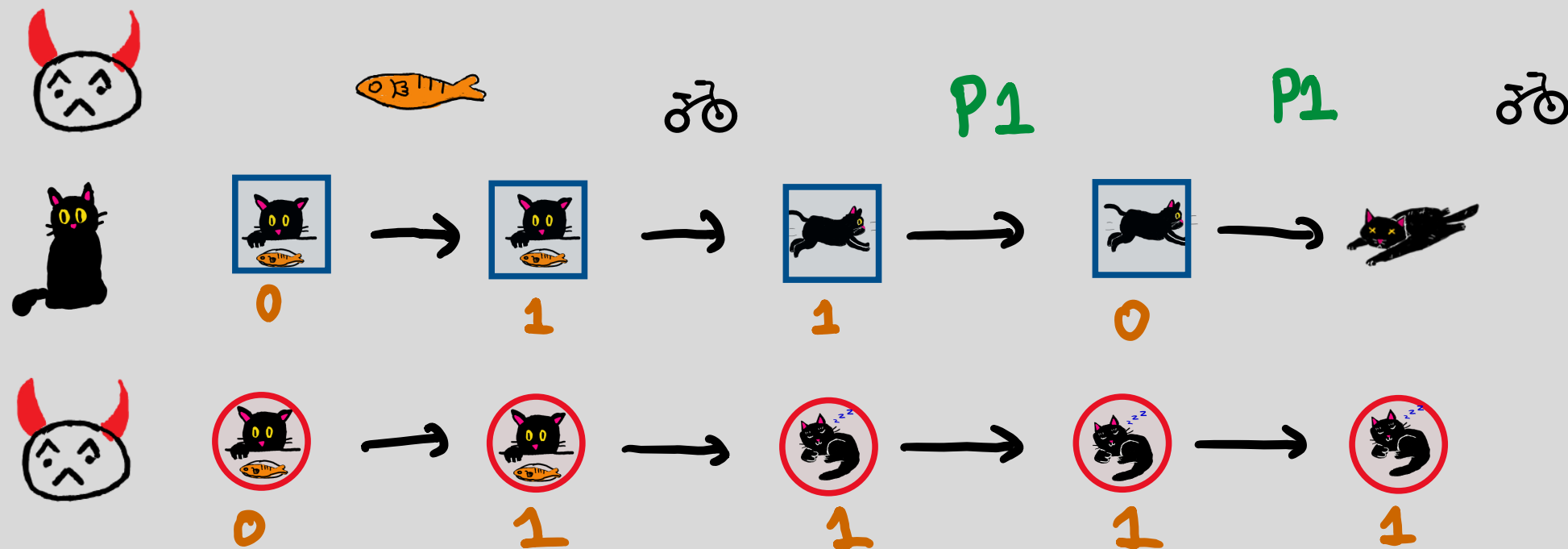


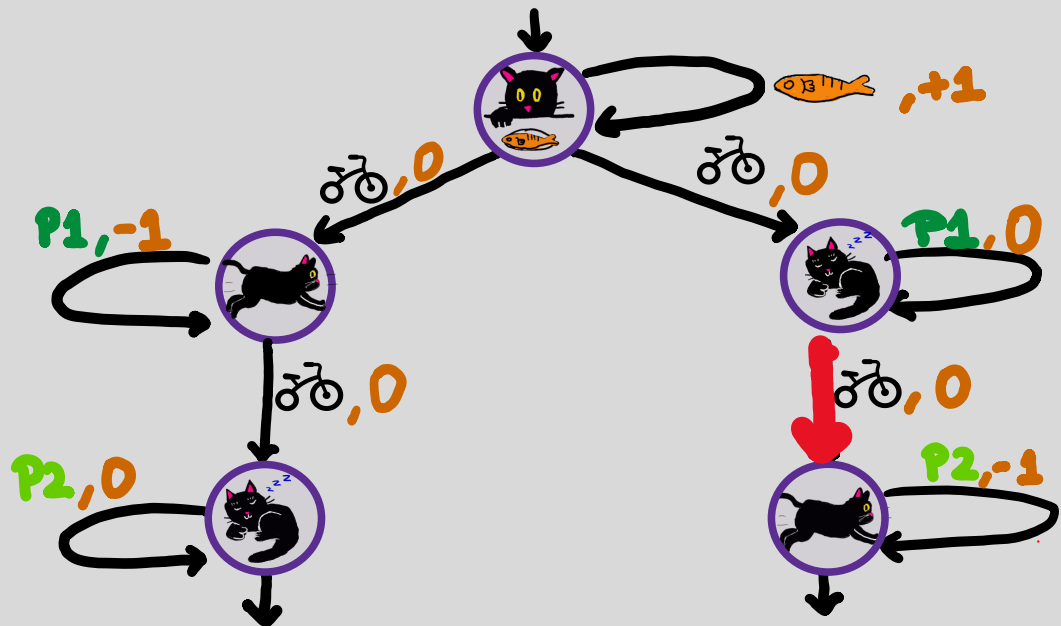
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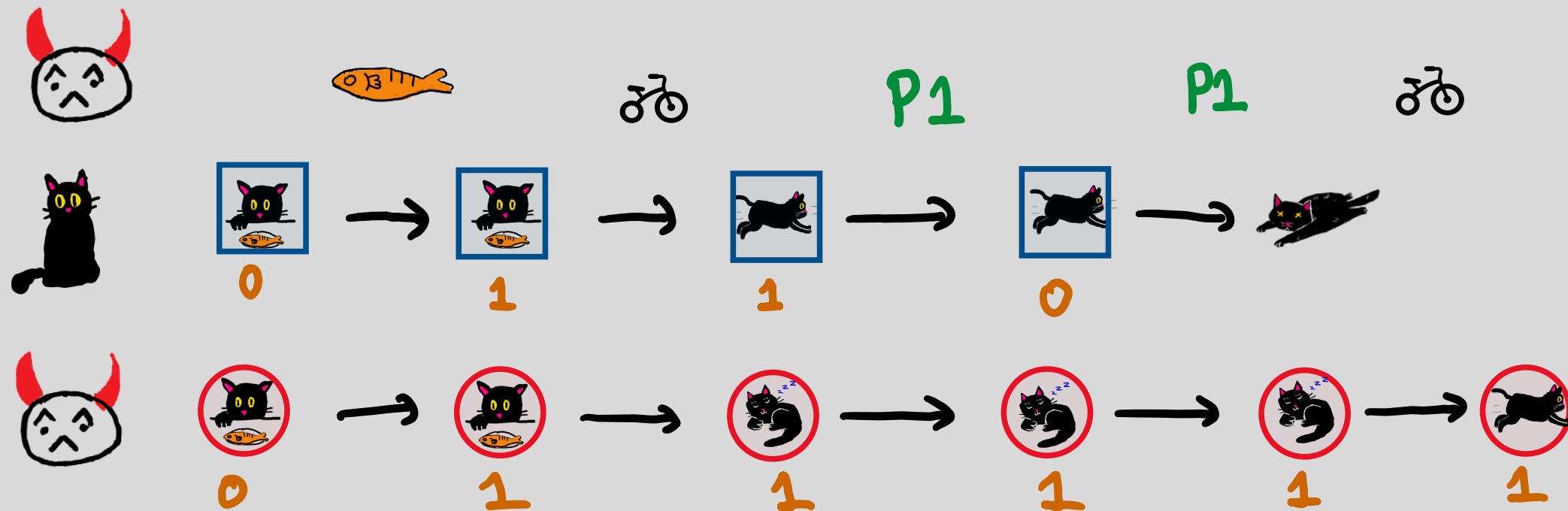


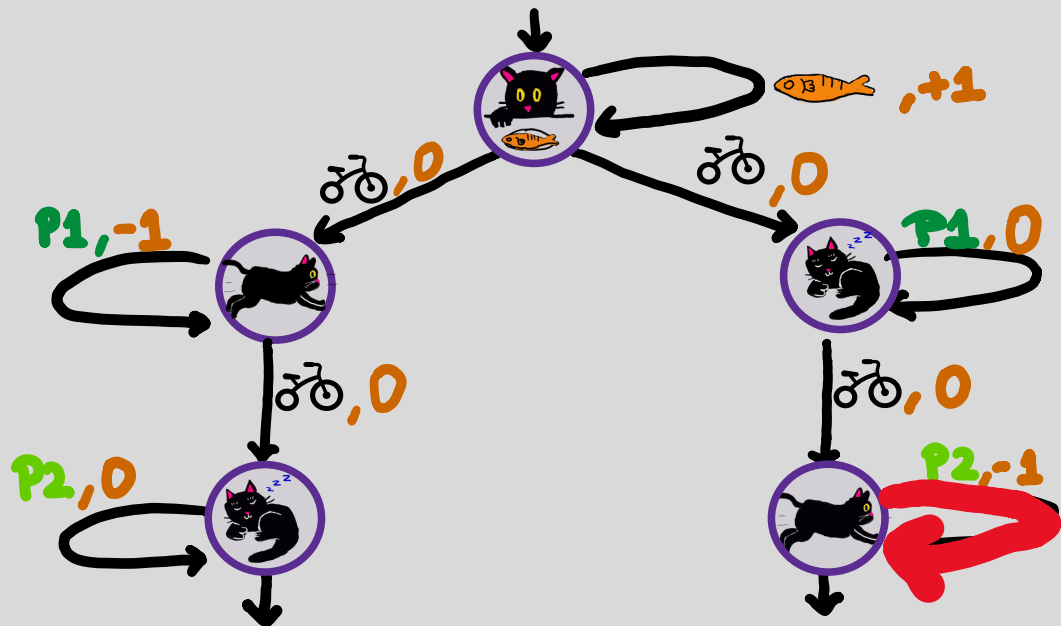
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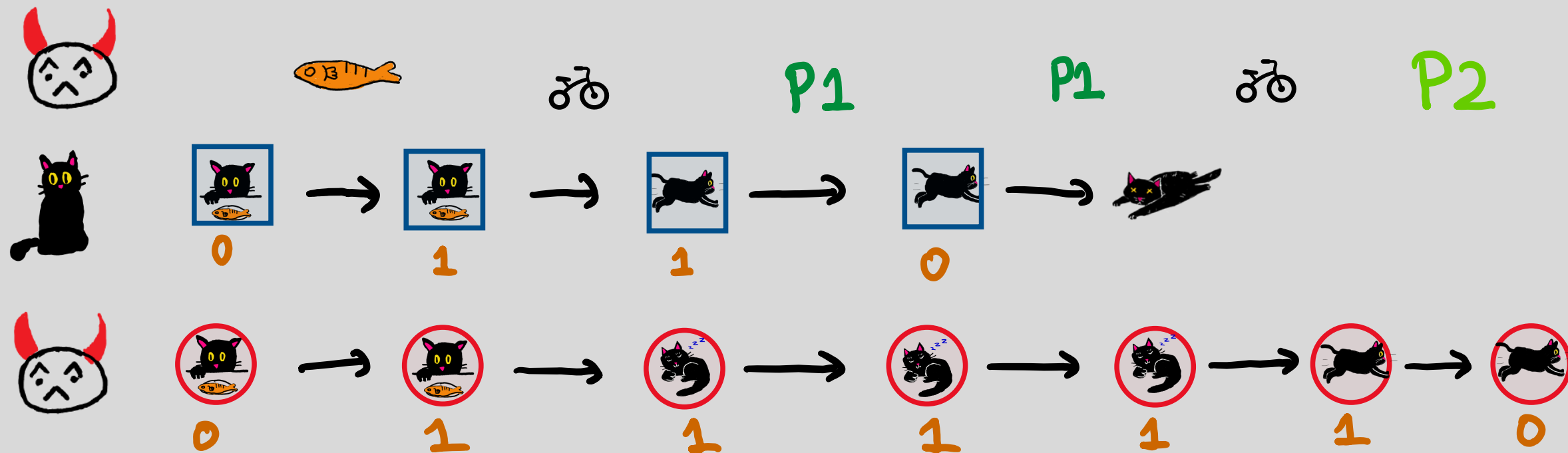


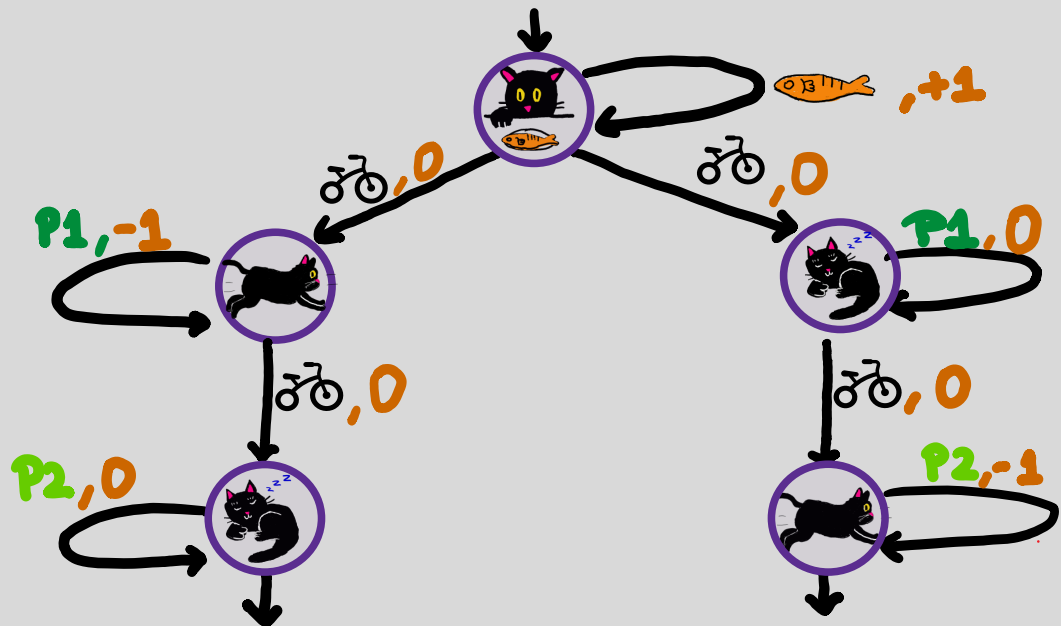
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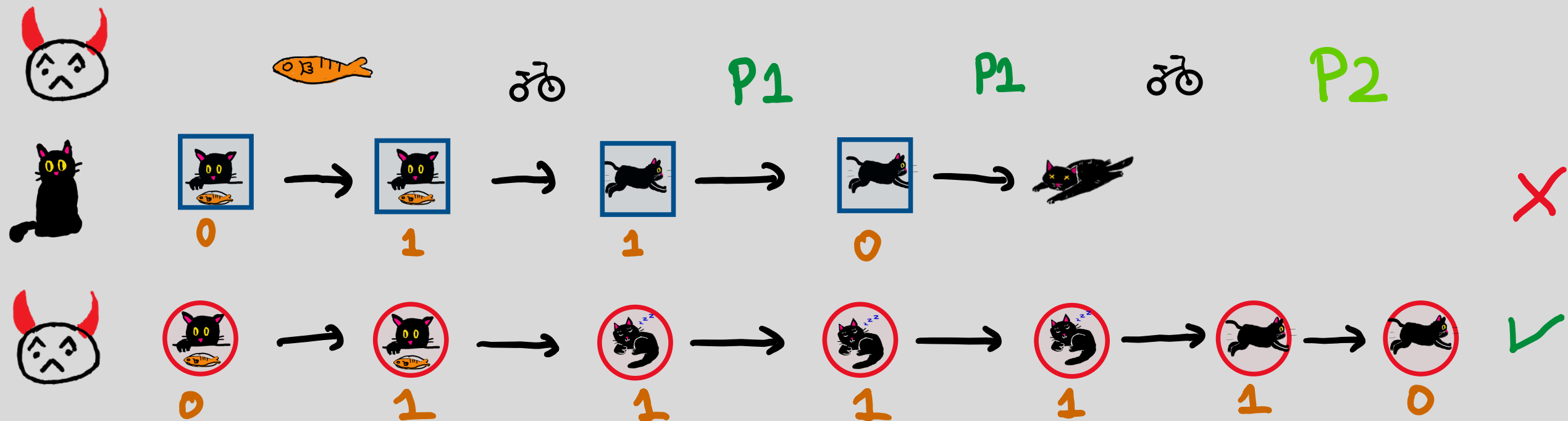


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HD Game:

Winning Condition for : If 's word is accepting, and 's run is rejecting.

Token Game:

Winning condition of : If 's run on  is accepting and 's run on  is rejecting.

Theorem [Boker, Lehtinen'22]



wins 1-token game $\Leftrightarrow$



wins HD-game

Theorem: Given an history-deterministic OCN,
there is a semilinear strategy for  that
is effectively computable.

Proof:

HD
Game



Token
Game



Simulation
Game

[Boker, Lehtinen'22]

[Our work!!]

Theorem [Hofman, Lasota , Mayr, Totzke '16]

Deciding the winner in the simulation game
between two OCNs can be done in
PSPACE, and the winning player has
semilinear strategies.

Proof:

HD
Game



Token
Game



Simulation
Game

Semilinear
Strategies

Proof:

HD
Game



Token
Game



Simulation
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Semilinear
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Token
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Simulation
Game

Semilinear
Strategies

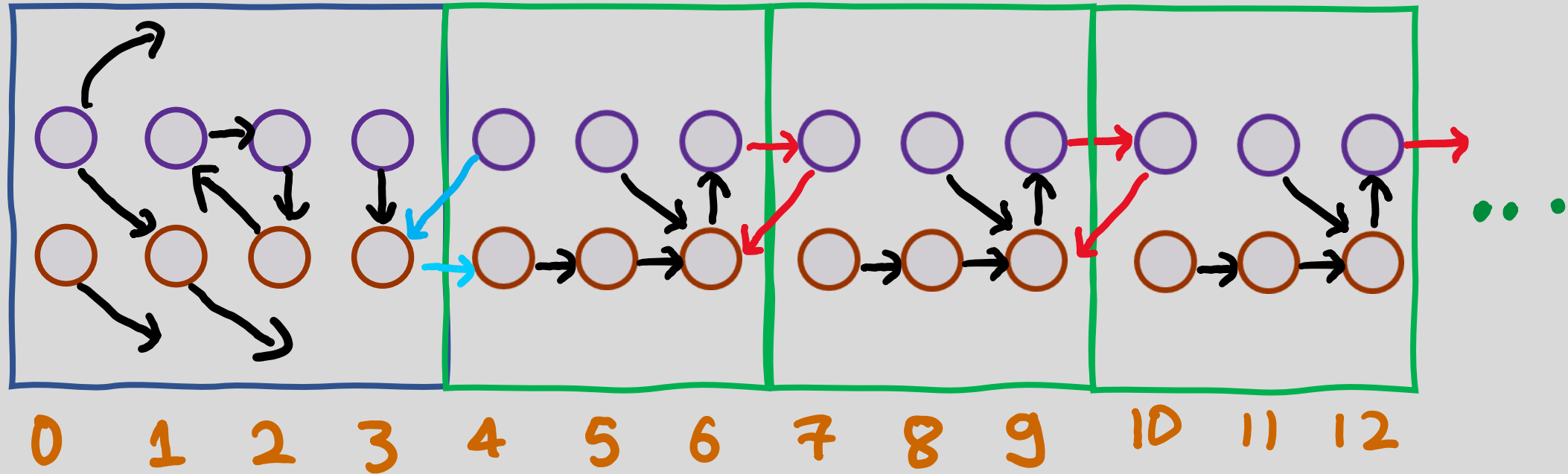


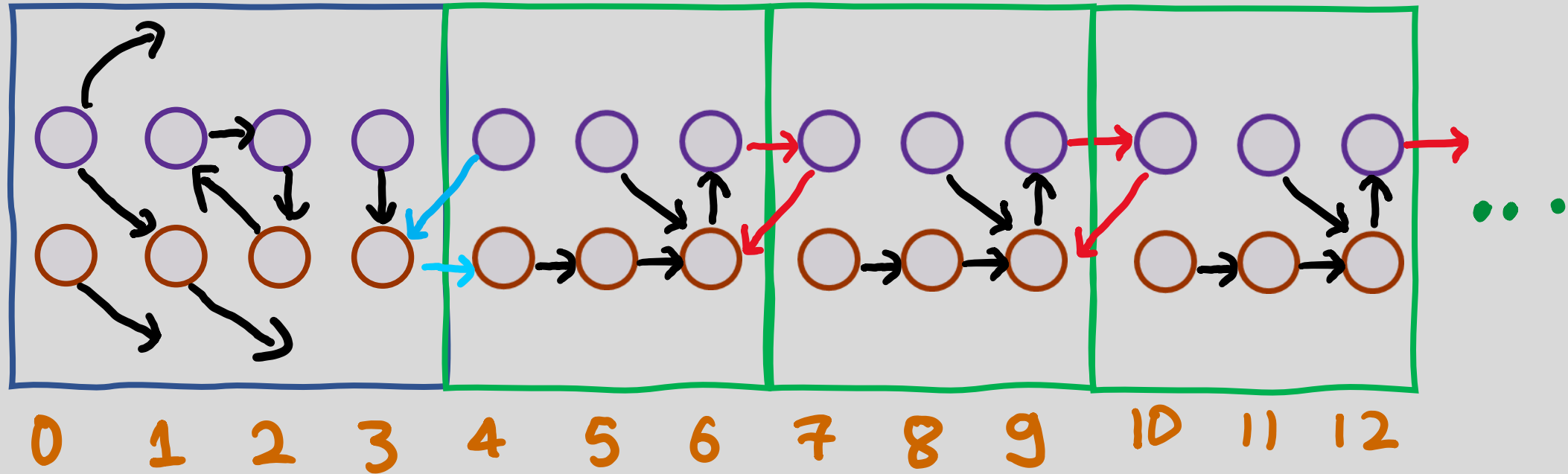
Semilinear
Strategies



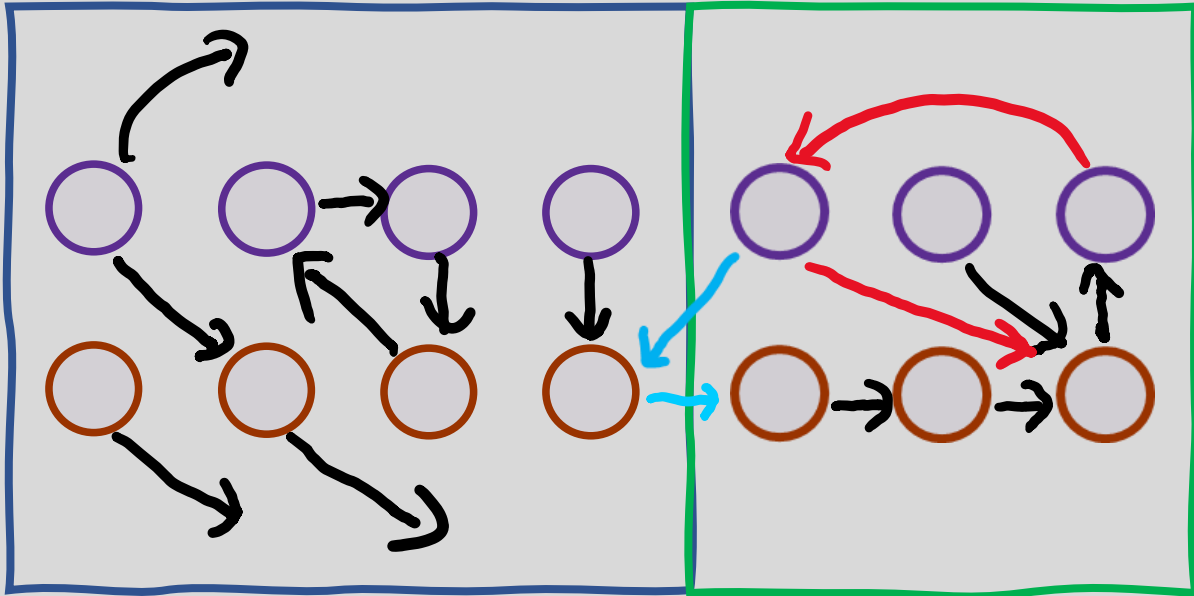
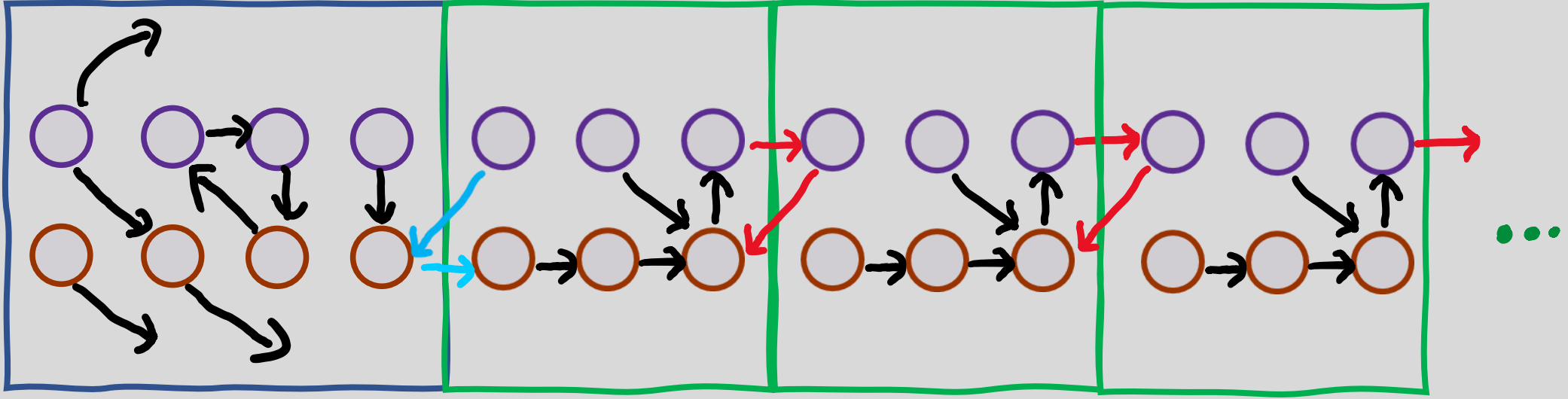
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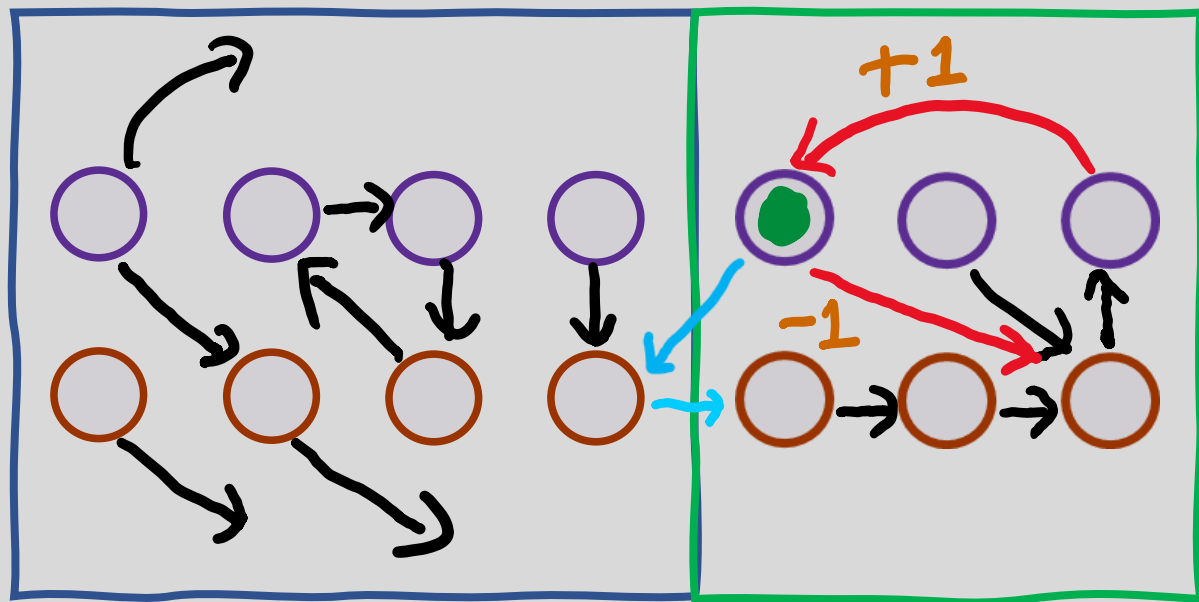
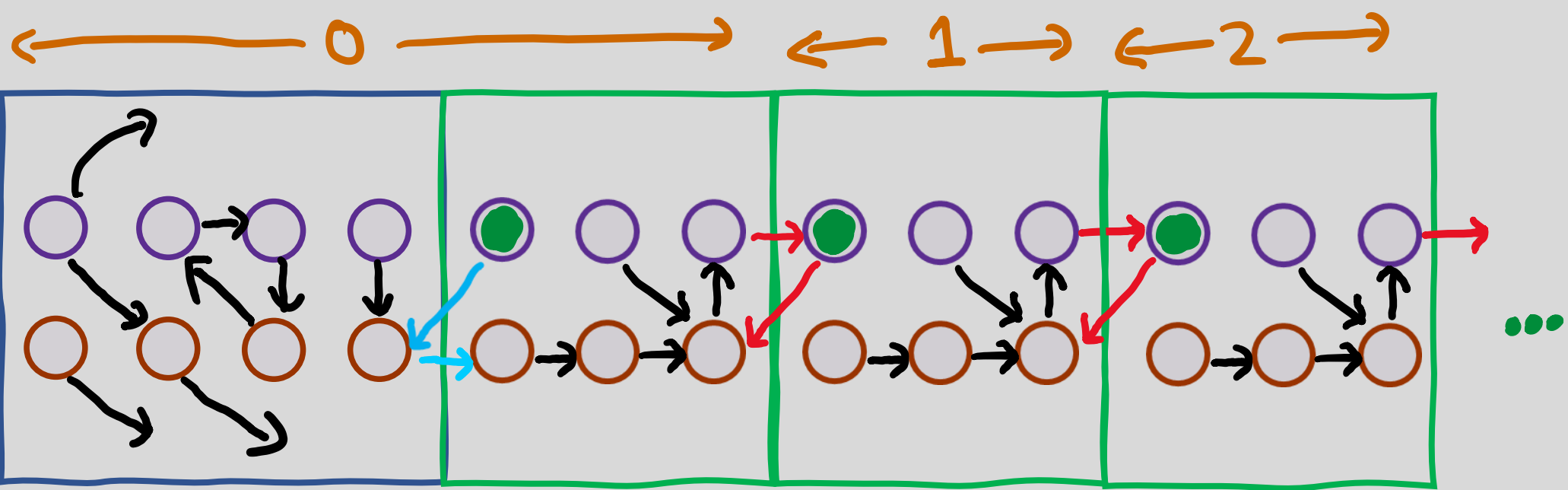
Theorem: Given an history-deterministic OCN,
there is a semilinear strategy for  that
is effectively computable.





Note that any run that follows 's winning strategy is accepting on all accepting words.





Theorem: Given an history-deterministic OCN,
there is a semilinear strategy for  that
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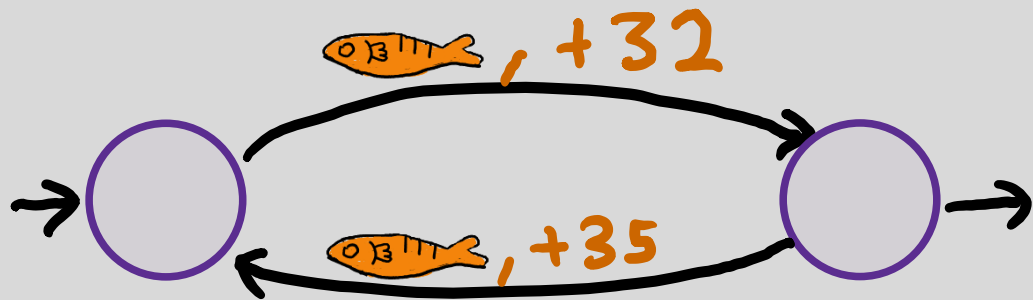
Corollary: Every history-deterministic OCN
can be converted to a language equivalent
deterministic OCA.

Theorem [Hofman, Lasota, Mayr, Totzke '16]

Deciding the winner in the simulation game between two OCNs can be done in PSPACE.

Theorem: Deciding whether a given OCN is history-deterministic is $PSPACE$ -complete, and $EXPSPACE$ -complete when the counters are encoded in binary.

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Theorem: Deciding whether a given OCN is history-deterministic is $PSPACE$ -complete, and $EXPSPACE$ -complete when the counters are encoded in binary.

Proof: Upper bound - Reduction to Simulation game

Lower bound - Reduction from Reachability game on OCN

History-Deterministic One-Counter Nets

► The resolvers in an HD-OCN are semilinear

Corollary: Every HD-OCN can be converted to a deterministic one-counter automaton.

► Complexity of checking history-determinism:

1. PSPACE-complete for unary encoding
2. EXSPACE-complete for binary encoding

3. History - Deterministic

One-Counter Automata

What about One-Counter Automata?

What about One-Counter Automata?



Checking for history-determinism:

What about One-Counter Automata?



Checking for history-determinism: Undecidable

What about One-Counter Automata?



Checking for history-determinism: Undecidable

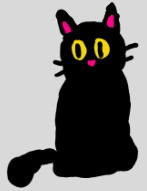


Do all HD-OCA admit language-equivalent DOCA?

What about One-Counter Automata?

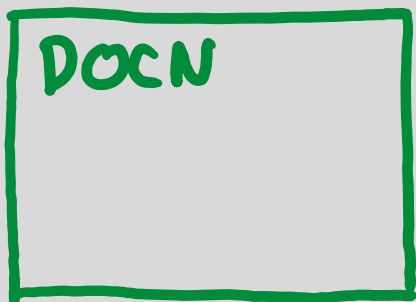


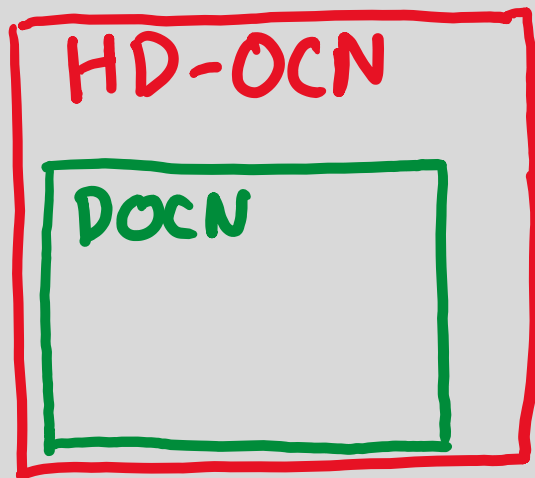
Checking for history-determinism: Undecidable

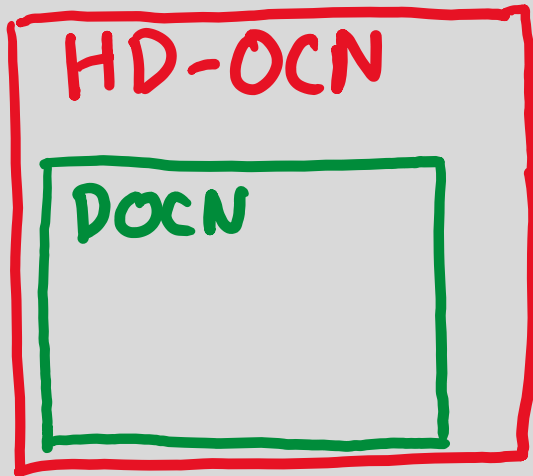


Do all HD-OCA admit language-equivalent DOCA?

Open. Semilinearity of strategies suffice.

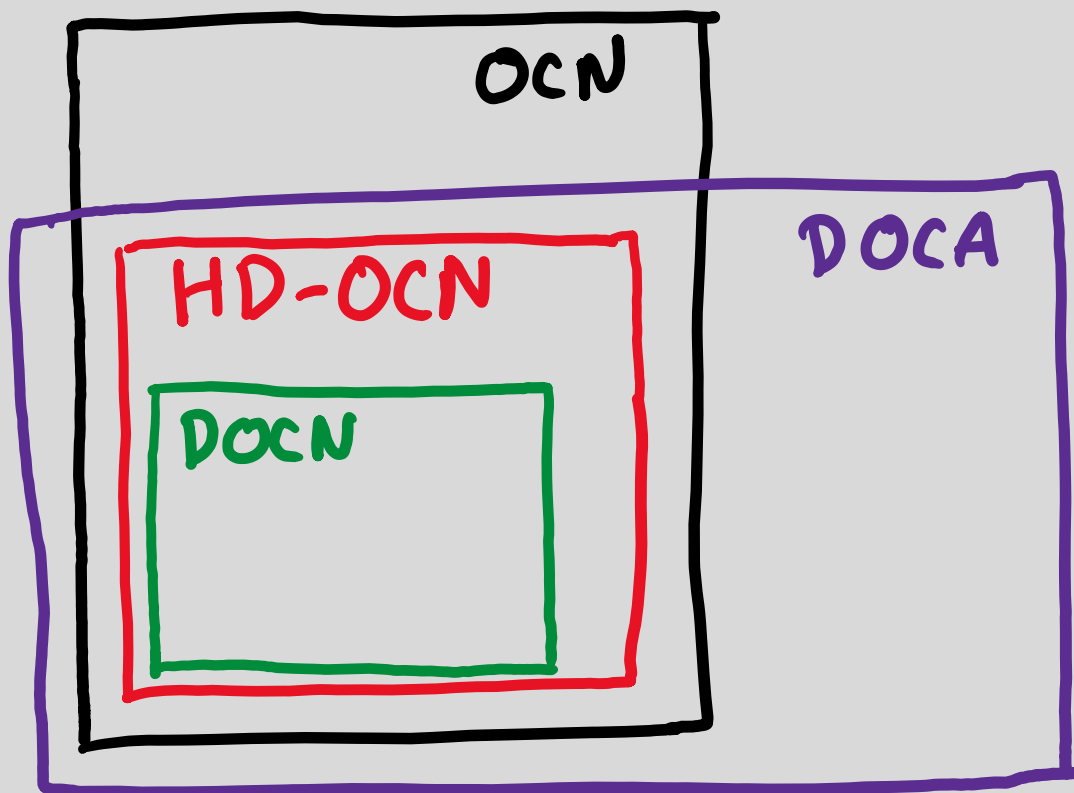




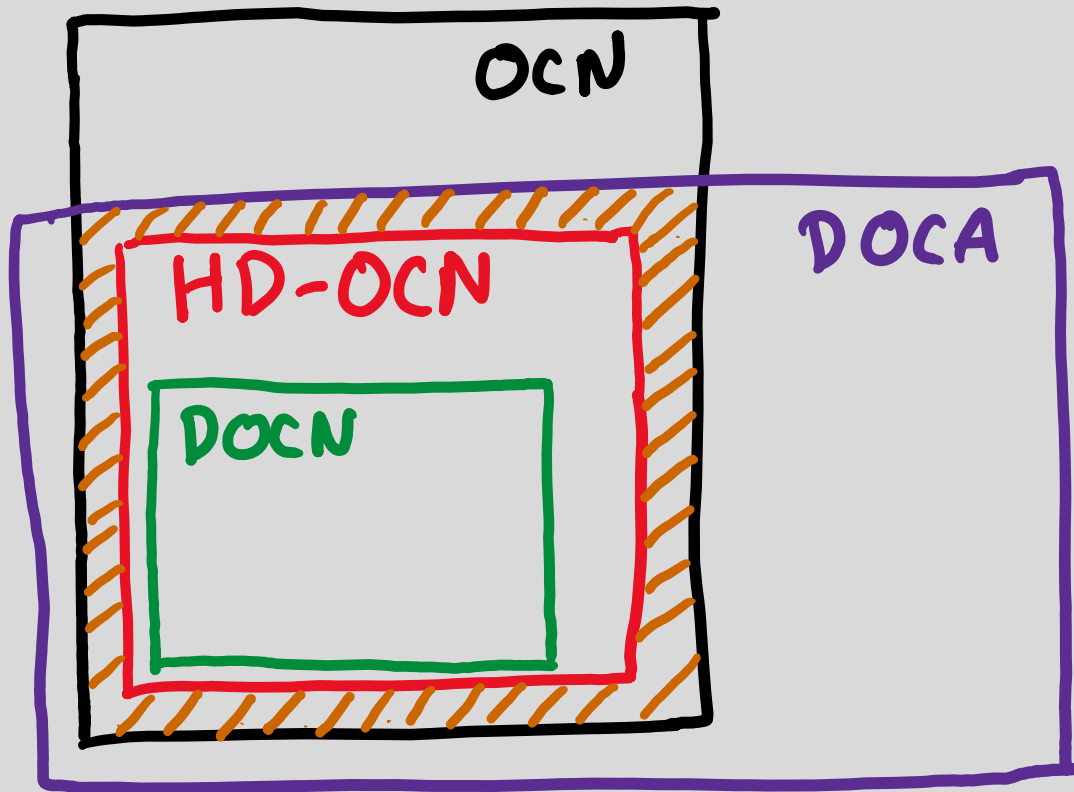


HD-OCNs are more expressive than DOCNs.

[S. Almagor, A. Yeshurun]
Private Communication



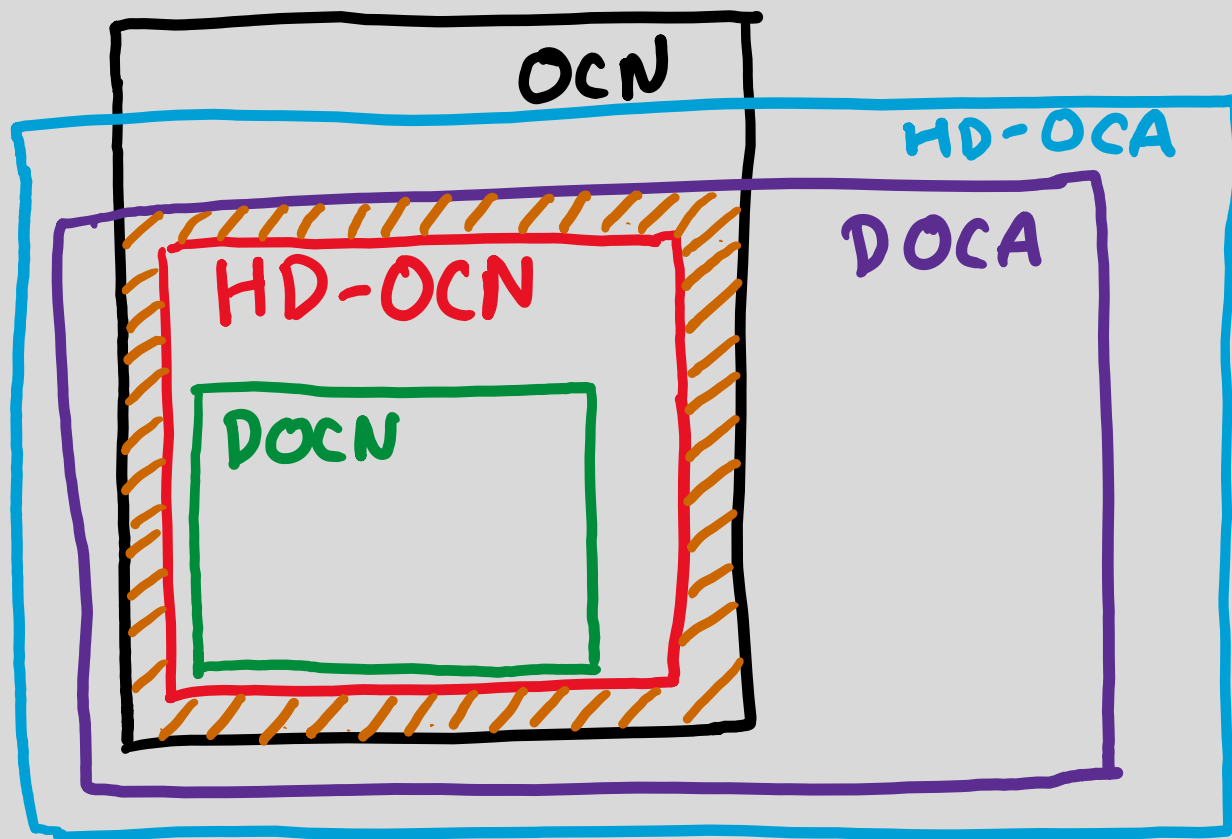
$$HD-OCN \subseteq OCN \cap DOCA$$



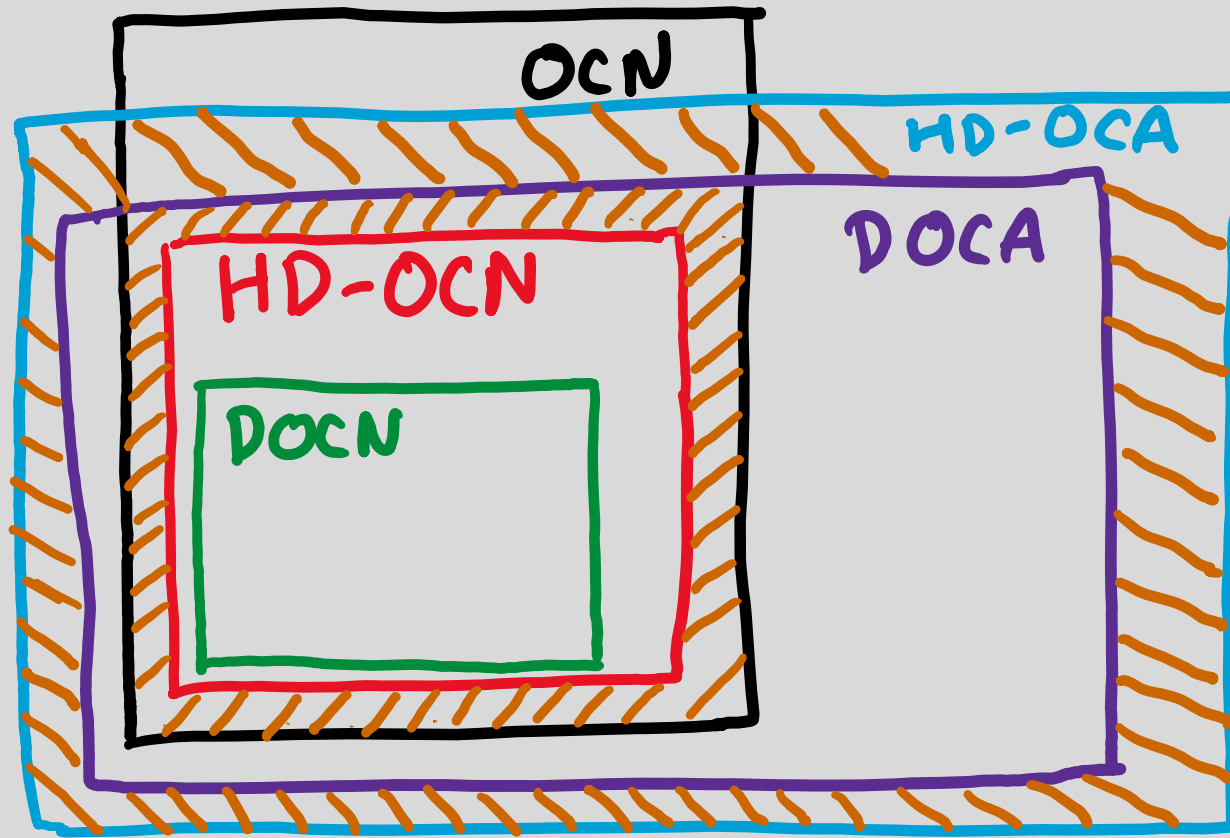
$$HD-OCN \subseteq OCN \cap DOCA$$

Is $HD-OCN = OCN \cap DOCA$?

Open.



Is $HD-OCA = DOCA$?



Is $HD-OCA = DOCA$?

Open

Conclusion



Every history-deterministic OCN has a language-equivalent DOCA.



Checking history-determinism for OCNs is PSPACE-complete.



Checking history-determinism for OCAs is undecidable.

Open Problems



Is $HD-OCA = DOCA$?



Is $HD-OCN$
 $= OCN \cap DOCA$?

